

Fiscal Rules and Discretion with Risk of Default*

Chiara Felli¹, Facundo Piguillem², and Liyan Shi²

¹LUISS

²EIEF and CEPR

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Abstract

It is widely believed that governments tend to overaccumulate debt, which gives rise to the need for fiscal rules. This paper studies the optimal fiscal and default rules when governments can default on their debt obligations. We build a continuous-time model that encompasses the standard rationale for debt overaccumulation: hyperbolic discounting and political economy frictions. In addition, governments are subject to taste shocks, which makes spending optimally random. Since shocks are private information, there is a trade-off between rules and discretion. We derive the optimal fiscal rules which are debt-dependent only when default is possible. Depending on the severity of the spending bias and the cost of default, the optimal fiscal rules range from strict debt limits, complemented by strong deficit limits, to the absence of all rules. In intermediate cases, debt-dependent deficit limits must be complemented with default rules, with some areas where default is banned and others where default is mandatory.

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1 Introduction

Over the past few decades, sovereign debt has substantially increased in many developing and advanced economies. As a result, fiscal rules have become increasingly prevalent (Yared, 2019). The driving force behind this wave of rules is the concern about debt sustainability and the implied risk of default. However, the implemented rules are in general ad hoc and not based on sound theories—and when they are, they mostly abstract from the interaction with risk of default.¹ How do fiscal rules change when a sovereign can default? Should there also be “default rules”? In this paper, we show that depending on the economic environment many possibilities can arise, ranging from replacement of fiscal rules with default rules to the imposition of constitutional borrowing limits with hard spending limits.

When analyzing fiscal rules, a question naturally arises: what is the underlying friction generating the need to impose rules? One of the commonly accepted reasons for imposing fiscal rules is rooted in political economy. In a nutshell, political turnover together with political polarization creates incentives for incumbents to overspend at the expense of future governments: there is a *spending bias*. This friction by itself is simple to deal with. If a rule-maker could perfectly observe current and future spending needs, a rule limiting the ability of governments to spend would easily ensure that only optimal spending decisions were possible. However, spending needs are affected by random and unpredictable events that render it necessary to endow policymakers with the discretion to optimally adjust spending and, thus, to smooth out the consequences of these shocks. Still, if the shocks were fully observable and contractible, a contingent fiscal rule using this information would again solve the problem. In reality, information is imperfect, but even when it is relatively precise, it is hardly contractible. This creates a meaningful trade-off between discretion, to allow governments to respond to shocks, and rules to prevent them from overspending.

Starting from this premise, there is a literature, originated by Amador, Werning, and Angeletos (2006), that analyzes the optimal trade-off between commitment and flexibility when agents discount the future quasi-hyperbolically. This approach has been extended by Halac and Yared (2014), who applied it to governments and interpreted the outcomes as fiscal rules. However, this literature abstracts from the possibility of debt repudiation. Thus, the prescriptions are only about spending or deficit limits and are independent of the level of debt. Moreover, the possibility of default brings about new dimensions to the trade-off between discretion and rules. Since this possibility increases welfare when the financial markets are incomplete (Dovis, 2018), many questions arise. Should default be restricted?

¹See Eyraud, Hodge, Ralyea, and Reynaud (2020). There are a few exceptions, e.g., Hatchondo, Martinez, and Roch (2015), Adam and Grill (2017) and Alfaro and Kanczuk (2017), which we describe in Section 1.1.

That is, should there be default rules? If so, in which scenarios? Does the possibility of default affect fiscal rules in the states in which the default option is not exercised?

To analyze this problem, we develop a continuous-time model with *spending-biased governments*. Consider an environment where spending needs are random. These genuine needs represent the real social value of spending, but they are observed only by the incumbent government. Thus, it is desirable to endow governments with some discretion to react to them. At every instant, with some probability, a change of government can occur: the incumbent is replaced by a new one who draws a new spending need. Governments are forward-looking, but they value the decisions made by other governments less. To be precise, the incumbent discounts any allocation chosen by any future government by a factor $\beta \leq 1$. This factor captures the extent of political polarization. If $\beta < 1$, political turnover generates a *spending bias* resembling hyperbolic discounting. Even though the needs are genuine, because of the spending bias, the incumbent has incentives to overstate them and overspend. Hence, imposing fiscal rules could be instrumental in restoring efficiency.

Governments can save and borrow using a noncontingent short bond, which is supplied by a continuum of risk-neutral international lenders. Since at any instant the government can default, the interest rate charged on the loans endogenously reflects the default risk. When the government defaults, it is excluded from the financial markets, as in [Eaton and Gersovitz \(1981\)](#), and it suffers a proportional loss in resources, as in [Arellano \(2008\)](#). This status does not need to be permanent. With some probability, the government regains access to the financial markets, and the resource loss vanishes. Despite the potential loss in output and future insurance, default could be welfare-improving, because it helps complete the markets. However, due to the spending bias, the incumbent's decisions may be inefficient, which creates another impetus for rules.

To design the optimal regulation, we study a mechanism-design problem. We take the perspective of a noncommitted, benevolent planner who chooses spending and default allocations subject to the truthful revelation of spending needs. We then show that these optimal choices can be implemented as a Markov equilibrium between current and future governments, when they are subject to not only *fiscal rules* but also *default rules*. Our perspective has two appealing features. First, the mechanism-design approach does not constrain the set of instruments available to the rule-writer. The sufficiency of both fiscal and default rules *arises endogenously*. Second, the planner's lack of commitment ensures that rules are *sustainable*: future rule-writers have no incentive to change them ex post.

Our first contribution is to show that default rules are necessary complements to the standard fiscal rules. Default rules can take many forms. They can imply mandatory default after a certain level of debt or forbid default when the debt level is not high enough. In

between, governments would be endowed with full discretion to default, depending on their spending needs. A special case is a hard borrowing limit (the intermediate area is degenerated) coupled with mandatory default beyond the limit. For instance, the constitution could state that any debt level above a certain percentage of GDP would be illegal.

To understand the intervention on both ends, it is important to keep in mind that the spending bias makes it unclear whether there is too much or too little default. One may think a myopic government would tend to default too much, but that is not necessarily the case. The default decision arises from comparing the present value of the nondefault benefits with the present value of the default costs. The government's "myopic" discounting affects both costs and benefits. Hence, the intertemporal distribution of costs and benefits could lead to either over-default or under-default. This last possibility arises when the probability of reentry into the financial markets is large enough. *Reentering is a benefit* that happens in a potentially distant future. Thus, an incumbent, mostly concerned about its own term, may not fully internalize it. A rule-writer with an undistorted, intertemporal view would consider this benefit and force the current government to default.

Our second contribution is to show how the possibility of default affects fiscal rules. The presence of default risk and the default rules also affect the fiscal rules in the states of nature in which default does not happen. For debt levels for which there is no default risk, we find that the optimal fiscal rule is of the threshold type, as in [Amador et al. \(2006\)](#), independent of the debt level. To be precise, let θ be the reported spending needs, then there exists a threshold θ^{s*} such that all governments reporting $\theta \leq \theta^{s*}$ are unconstrained on their spending decisions, while all those reporting $\theta > \theta^{s*}$ must choose the same spending as a government reporting θ^{s*} . This optimal fiscal rule can easily be implemented with a spending cap or a deficit limit. In other words, θ^{s*} separate the areas between those governments that are endowed with discretion, low θ types, and those that are committed and must abide by the rule, high θ types.

For debt levels for which default risk is strictly positive, additional elements start to act. First, we show that the optimal fiscal rule is still of the threshold type but now *dependent on debt*. Let b be the government's financial position, so that $b < 0$ is debt, then there exist a threshold $\theta^s(b)$ such that all types $\theta \leq \theta^s(b)$ are unconstrained, while all $\theta > \theta^s(b)$ spend no more than $\theta^s(b)$. Again, this allocation can be implemented with either a spending cap or a deficit limit, but now the fiscal rule can be tightened or loosened as debt rises.

To uncover the interaction between the risk of default and fiscal rules, one must consider several factors. First, the possibility of default may render the spending/deficit constraint innocuous, because the planner may prefer a high spending type to default rather than constraining its spending choice. This indeed happens when political polarization is sufficiently

low (β is close to 1). In this case, the planner optimally chooses that all types $\theta \geq \theta^d(b)$ must default. Since $\theta^d(b) \leq \theta^s(b)$, there is no need to impose spending limits. On the equilibrium path this outcome would look like a roughly constant deficit or spending limit as long as there is no default risk, and all these restrictions would be lifted as soon as the default risk becomes positive. The task of imposing discipline is allocated entirely to the market.

Second, if the planner deems it optimal not to default, i.e., $\theta^d(b) > \theta^s(b)$, the fiscal rule imposed in the absence of default risk must be modified. To this end, they must consider the effect of the rule on the interest rate, through the risk premium, and on the government's incentives to default. The interest-rate effect brings out some elements reminiscent of Halac and Yared (2018). An interest-rate hike is good for imposing discipline on myopic governments, but unlike in Halac and Yared (2018), it has a negative income effect, since interest payments are transfers to foreign lenders. As a result, as debt grows, so does the risk premium; therefore, the planner chooses to reduce discretion by tightening the spending or deficit limits. In addition, by manipulating the spending threshold, the planner alters the default threshold, which in turn changes the government's incentives to overspend.

Finally, our paper bridges the theoretical and quantitative literature, analyzing all these instruments simultaneously. Partial answers to these questions can be found in previous works. Some works are highly theoretical and stylized, while others focus on fully quantitative models incorporating one policy instrument at a time. Our framework provides sharp theoretical characterizations of the optimal rules under a rich set of political and economic environments, while allowing us to evaluate its quantitative implications meaningfully.

To this end, we calibrate the model economy to characterize three types of economy: 1) one with a low debt capacity and a risk premium highly sensitive to debt accumulation, which we call the Greece-like regime; 2) one with a high debt capacity and a sensitive risk premium, that we call the Italy-like regime; and 3) one with a high debt capacity combined with a mildly sensitive risk premium. Here we emphasize two important findings. First, the interaction between fiscal rule and default rule is sizeable. Absent any fiscal rule, the optimal default rule follow closely the governments' preferred decisions; when complemented with a fiscal rule, the intervention greatly modifies the governments' choices. Second, the combination of fiscal and default rules implies a very large debt capacity. This outcome resembles an otherwise seemingly unsustainable debt burden, together with extreme austerity measures. These two elements together, which are on occasions regarded as irrational, arise naturally as optimal rules in our setup.

The paper is organized as follows. Section 1.1 reviews the literature. Section 2 describes the environment under which we generate our results, including the equilibrium in the absence of any rule. Section 3 contains our main results, delivering both the optimal fiscal

and the optimal default rules. Section 4 generalizes the results. Section 5 evaluates the quantitative implications applied to selected European countries. Section 6 concludes.

1.1 Literature Review

We relate to three strands of literature: on dynamically inconsistent preferences and their relation to political economy; on the optimal trade-off between commitment and flexibility; and the rich, growing body of work on sovereign default.

The literature closest to our work was originated by [Amador et al. \(2006\)](#), who analyze the optimal trade-off between commitment and flexibility when agents discount the future quasi-hyperbolically.² The central premise in this literature is that agents (or governments) are tempted to overspend, so ideally it would be optimal to limit their possibilities to accumulate debt. What makes the problem nontrivial is that agents are subject to random spending needs, which are either not observable or not contractible. Thus, it would be desirable to endow them with some discretion. More recently, this approach has been extended by [Halac and Yared \(2014\)](#), allowing for persistence shocks when preferences are logarithmic, and [Halac and Yared \(2018\)](#), who consider the endogeneity of the interest rate. With respect to them, we cast the problem in a continuous-time framework, and we add the possibility of default. We show that the optimal fiscal rule can be debt-dependent and is complemented by default rules.

We also build on the rich literature analyzing environments with quasi-hyperbolic discounting, originated by [Strotz \(1955\)](#) and augmented by [Laibson \(1997\)](#). These environments are in general analyzed in discrete-time frameworks.³ However, as [Chatterjee and Eyigungor \(2016\)](#) pointed out, this approach generates many technical challenges when the economic agents are subject to borrowing limits. In particular, all Markov equilibria generate discontinuous decision rules. Since default decisions imply de facto borrowing limits, we avoid this difficulty by modeling decisions in continuous time, similarly to [Harris and Laibson \(2013\)](#). They consider a limiting behavior which they termed “instant gratification.” Our modeling strategy is instead more closely related to [Cao and Werning \(2016\)](#).⁴ They analyze the savings behavior for a more general “disagreement index,” while we maintain it constant, interpreting it as political polarization. With respect to them, we allow for the possibility of default that endogenizes both the interest rate and the type-contingent borrowing limits.

Combining these two approaches allows us to build a bridge between the highly theoretical

²See [Ambrus and Egorov \(2013\)](#) for corrections to the original results and [Athey, Atkeson, and Kehoe \(2005\)](#) for an alternative approach with endogenous time inconsistency.

³See for instance [Krusell and Smith \(2003\)](#) and [Cao and Werning \(2018\)](#).

⁴For another application of quasi-hyperbolic discounting in continuous time see [Laibson, Maxted, and Moll \(2020\)](#).

and the quantitative analysis, looking for more precise answers to actual rules that should be implemented in real life. By imposing additional structure in the friction generating over-accumulation of debt, the model allows for a meaningful mapping to the data. To be precise, it maps neatly to the standard political economy models á la [Persson and Svensson \(1989\)](#), [Alesina and Tabellini \(1990\)](#), and [Battaglini and Coate \(2008\)](#). Thus, we can decompose the hyperbolic discounting factor into *political turnover* and *polarization*. This decomposition is along the line of recent quantitative work by, for example, [Azzimonti, Battaglini, and Coate \(2016\)](#), who assess the effect of imposing a balanced budget.

Finally, we contribute to the theoretical and quantitative literature analyzing sovereign default. We build on the seminal contributions of [Eaton and Gersovitz \(1981\)](#) and [Arellano \(2008\)](#). However, our modeling approach of default is more closely related to [Bornstein \(2020\)](#), who analyzes the case of exponential discounting. Beyond the abundant positive, primarily quantitative, literature studying sovereign default, there are few studies focusing on normative issues. [Dovis \(2018\)](#) introduces private information and lack of commitment to debt repayment as we do, but all agents discount the future consistently. Thus, he derives prescriptions for optimal lending agreements, not fiscal rules. Instead, [Hatchondo et al. \(2015\)](#) analyze how committing to future decisions, imposing fiscal rules, could improve current outcomes. With respect to them, we introduce the present bias and consider the interaction between fiscal and default rules, even though we do not incorporate long-term debt.⁵ To the best of our knowledge, [Adam and Grill \(2017\)](#) is the only paper studying the possibility of incorporating default rules. They study a Ramsey equilibrium with perfect information and geometric discounting. Nevertheless, they find that when default is costly, it is only optimal to default when shocks are of “rare disasters” type. Finally, [Alfaro and Kanczuk \(2017\)](#) analyze a discrete-time environment similar to ours. They build a quantitative model and evaluate the welfare properties of some selected rules, whereas we derive and characterize theoretically what rules are optimal for different environments.

2 Model

This section develops a dynamic model incorporating the standard frictions of spending bias and default choice. We first briefly describe the model’s fundamentals, then we characterize the equilibrium without fiscal rules.

⁵See also [Chatterjee and Eyigungor \(2015\)](#) and [Hatchondo, Martinez, and Sosa-Padilla \(2016\)](#) for normative prescriptions to improve lending contracts.

2.1 Environment

Time is continuous and infinite, $t \in [0, \infty)$. At every instant $t \geq 0$, the economy is governed by an incumbent government. A *political turnover* event occurs with Poisson arrival rate λ : the incumbent government loses power and is replaced by a new one. Each incumbent government receives an exogenous source of tax revenue τ and faces a spending choice g . Different governments attribute different values to their spending needs. This value is determined by their “taste” type θ , which is an *i.i.d.* random variable, drawn from a bounded set $\Theta \equiv [\underline{\theta}, \bar{\theta}]$ according to the cumulative distribution function $F(\cdot)$ and expected value $\mathbb{E}[\theta] = 1$. The change in preferences can be interpreted as arising from the underlying constituency’s opinions on the social value of spending, changing over time and determining the alteration of the country’s stance on fiscal policy.⁶ The preferences for spending flows are

$$\theta u(g),$$

where $u(\cdot)$ is strictly increasing and strictly concave, $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Thus, types with high θ experience a larger marginal utility from spending than low types.

Spending bias. All governments, whether incumbent or opposition, discount the future exponentially at rate ρ . However, the incumbent government values spending by future governments less. To be precise, every unit of spending transforms into one unit of consumption when the government is in power, but it delivers only $0 < \beta < 1$ units of consumption when the government is out of office. To simplify the analysis, we recast this political friction in terms of utility. Namely, we assume that governments discount utility by the extra-term β whenever they are not in power. Thus, introducing the discount term β is a reduced-form way of capturing disagreement within a country over the composition of public spending, rather than over its level. For this reason, we refer to β as the *political polarization* parameter. In Section 2.2, we explain how β shapes the outcomes and further discuss our interpretation, providing additional ones.

Information friction. The realization of spending needs θ is privately observed by the government, which renders it impossible to write contracts contingent on it. The critical part of this assumption is that θ is noncontractible. There are many reasons why the government

⁶One possible interpretation is that preferences vary in response to the business cycle. For instance, Amador et al. (2006) show that if utility is exponential, taste shocks are equivalent to income shocks. Another interpretation is that demographic changes in the constituency’s composition or power struggles between different parties induce a preference shock. See, for example, the entrepreneur-worker conflict in Azzimonti, de Francisco, and Quadrini (2014). Nevertheless, in Section 4.2, we provide an environment where we make the revenue τ random.

could be better informed about its spending needs. Moreover, even when the shocks were observable or ex post verifiable, they may not be contractible. For instance, one may think that it would be politically infeasible to write a policy rule that constrains a specific political party. Still, the shocks are assumed to *represent genuine spending needs*. This creates a meaningful trade-off between discretion, to smooth out the shocks, and rules to contain the spending bias.

Default. Let b denote the value of outstanding government bonds. If $b > 0$, the government saves in assets; while if $b < 0$, it is in debt. At any instant, the government can default on its debt obligations, upon which it is excluded from financial markets, as in [Eaton and Gersovitz \(1981\)](#). While in default, tax revenues are reduced to $\kappa\tau$, with $\kappa \in [0, 1]$. This captures output losses due to financial exclusion, as in [Arellano \(2008\)](#). Financial access is regained at Poisson rate $\phi \geq 0$: upon reaccess previously defaulted debt is fully discharged, and the government returns to the market with a zero-debt position. While how much debt is discharged for defaulting countries could be important, we show in [Section 4.1](#), where we extend the analysis to partial discharge, that it has a minor qualitative impact.

Let $\delta(\theta, b) \in \{0, 1\}$ denote the default decision for a government of type θ at debt position b , which takes a value 1 if it defaults and 0 otherwise. Then,

Definition 1 (Allocation). An allocation $\{g(\theta, b), \delta(\theta, b)\}_{\theta \in \Theta, b \in \mathbb{R}}$ specifies the government spending and default decisions for all types at all debt levels.

Interest rate. There is a continuum of competitive risk-neutral lenders with access to risk-free rate $r_f \leq \rho$. The risky interest rate charged to governments adjusts for the expected default probability:

$$r(b) = r_f + \lambda \mathbb{E}[\delta(\theta, b)]. \quad (1)$$

Given the interest rate, the debt accumulation process follows

$$\dot{b}(\theta, b) = r(b)b + \tau - g(\theta, b). \quad (2)$$

It is worth mentioning the lending-market features that lead to equation (1). Despite the government's type θ being private information, the lenders could observe its actions and recover the true type. Thereby the lenders could potentially charge an interest rate depending not only on how much debt the government has accumulated but also on its specific type. However, the equilibrium interest rate depends *only* on the debt level b , not on the type θ .

To understand why, note that because time is continuous the accumulation process in equation (2) is smooth, and the interest-rate adjustment is instantaneous. Consider a borrower that has accumulated enough debt to be on the verge of default. The lenders would not extend additional funds to her, because the debt would be defaulted for sure. Alternatively, one may think that lenders would charge an infinitely high interest rate, preventing any borrowing. Thus, at the default threshold, further debt accumulation does not occur, i.e., $\dot{b}(\theta, b) \geq 0$. Default happens only when there is a discontinuous jump in θ .⁷ For this reason, the risk premium is the jump probability λ times the expected default rate of new types, $\mathbb{E}[\delta(\theta, b)]$. Still, the default probability depends on debt because, even if the current type would not default, by accumulating debt the current agent enlarges the set of future types who would default. By charging the appropriate interest rate, the lenders can make sure that the current government will not default, but they remain afraid that the future, still unknown, government would default at the given level of debt.

To fine tune the business-cycle properties of spending, in Section 4.2 we consider an extension where τ follows a *Browning motion, which is contractible*. Since Brownian motions move continuously over time, they *do not have a direct impact on default*. For instance, if we were to abstract from the “jump” θ shock and allow for only the Brownian motion, there would be no default on equilibrium, and the risk premium would be zero at all debt levels, except the default threshold where it would be infinity. For this reason, we develop the main results of the paper assuming that τ is constant and leave the implications of additional business-cycle features to Section 4.2.

Our choice of a continuous-time setup is not arbitrary—it is a key element that helps us overcome technical challenges. We just mentioned how this choice simplifies the default decision. In addition, the possibility of default generates endogenous “wealth limits” that, in combination with spending-biased agents, render the framework intractable. As [Chatterjee and Eyigungor \(2016\)](#) show, there would be no equilibrium in a discrete-time environment with continuous decision functions. The jumping actions that characterize the solutions in discrete time are no longer present in continuous time, which allows us to focus on equilibria that are differentiable almost everywhere.

⁷Alternatively, the interest rate schedule can be written as

$$r(\theta, b) = \begin{cases} r_f + \lambda \mathbb{E}[\delta(\theta, b)], & \text{if } \delta(\theta, b) = 0 \\ \infty, & \text{if } \delta(\theta, b) = 1. \end{cases}$$

[Bornstein \(2020\)](#), which studies a continuous-time version of the sovereign default model by [Arellano \(2008\)](#), also notes that it is critical to have stochastic jumps in order to observe defaults on the equilibrium path.

2.2 Rules-Free Equilibrium

Before proceeding to the analysis of optimal rules, it is instructive to discuss the equilibrium in the absence of rules. Moreover, a slight modification of this equilibrium, adding the constraints implied by the rules, is the benchmark used in the implementation. Let $w^j(\theta, b)$ be the value function of an incumbent with spending needs θ and financial position b and is in state $j \in \{n, d\}$, where d stands for default and n for nondefaulting status. Let $v^j(\theta, b)$ be the analogous value function from the perspective of a subject that values all governments' decisions equally, i.e., as if $\beta = 1$. When the economy is not in default, these two value functions solve the following system of Hamilton-Jacobi-Bellman (HJB) equations:

$$\rho w^n(\theta, b) = \max_g \{ \theta u(g) + (r(b)b + \tau - g)w_b^n(\theta, b) \} + \lambda (\beta \mathbb{E}[v(\theta', b)] - w^n(\theta, b)) \quad (3)$$

$$\rho v^n(\theta, b) = \theta u(g^*) + (r(b)b + \tau - g^*)v_b^n(\theta, b) + \lambda (\mathbb{E}[v(\theta', b)] - v^n(\theta, b)), \quad (4)$$

where $\mathbb{E}[v(\theta', b)]$ also embodies the expectation over future default decisions. Thus, if default were not possible $\mathbb{E}[v(\theta', b)] = \mathbb{E}[v^n(\theta', b)]$. To understand these equations, it is useful to start by assuming that default is not possible. Equation (3) makes clear that as long as $\lambda = 0$, this is a standard HJB equation for a continuous-time savings problem. When $\lambda > 0$, the political friction starts to play a role. The last term in equation (3) captures the effect of turnover. With arrival intensity λ , the incumbent loses its position, which implies a loss in value of $-\lambda w^n(\theta, b)$, and it is replaced by a new government, generating a value $\lambda \beta \mathbb{E}[v(\theta', b)]$. Here, two components are important. First, the future government's spending needs are not yet known; that is why there is an expectation over the future θ' . Second, and more importantly, the incumbent discounts ex ante the continuation value by the additional factor β .

For each θ , the continuation value satisfies the HJB represented by equation (4). It is clear from it that after the incumbent loses power, it discounts all future allocations at the same rate ρ , independently of the identity or type of the eventual government. The current government does not care who will be in power, as long as it is not itself. All non-me governments are equally discounted by β . Equation (4) also makes clear that the incumbent takes as given that future governments will spend (and accumulate debt) following their own optimal choices. That is the reason for the presence $g^* = g^*(\theta, b)$. The incumbent correctly assesses that, when it is not in control, whichever government in power would maximize its own utility.

In the absence of default, equations (3) and (4) would characterize the equilibrium. Instead, when default is possible, there is an additional decision, and the continuation value must take into account that future governments may default. Let $\delta^A(\theta, b)$ be an indicator

function denoting when a government defaults, so that:

$$\delta^A(\theta, b) = \begin{cases} 1, & \text{if } w^n(\theta, b) < w^d(\theta) \\ 0, & \text{if } w^n(\theta, b) \geq w^d(\theta). \end{cases} \quad (5)$$

Note that, as discussed in Section 2.1, even though default could happen at any instant, on the equilibrium path it only happens in the next instant after a change of government. This happens because the lenders can observe, or uncover, the incumbent's true type and would never lend an amount that would lead with certainty to default. Thus, the continuation value is given by:

$$v(\theta, b) = (1 - \delta^A(\theta, b))v^n(\theta, b) + \delta^A(\theta, b)v^d(\theta).$$

It remains to describe what is the value functions' evolution when in default. Recall that when a government defaults, it is forced into financial autarky and suffers a loss of resources, which leaves only $\kappa\tau$ to spend. The analogous to (3)-(4) are:

$$\rho w^d(\theta) = \theta u(\kappa\tau) + \phi (w^n(\theta, 0) - w^d(\theta)) + \lambda (\beta \mathbb{E}[v^d(\theta')] - w^d(\theta)) \quad (6)$$

$$\rho v^d(\theta) = \theta u(\kappa\tau) + \phi (v^n(\theta, 0) - v^d(\theta)) + \lambda (\mathbb{E}[v^d(\theta')] - v^d(\theta)). \quad (7)$$

One important difference between (3)-(4) and (6)-(7) is that optimization is no longer possible. Since countries cannot participate in the financial markets, it is impossible to smooth out spending. For the same reason, the HJBs in (6)-(7) do not display the derivative with respect to b . Both equations also make clear that when a government has the chance to reenter the financial markets, it makes it for sure. This is because it can return with zero debt as the terms $w^n(\theta, 0)$ and $v^n(\theta, 0)$ make clear. In Section 4, we relax this assumption and show how the default HJBs must be modified. Finally, notice that the political friction parameter β enters in the default status in the same way as in (3)-(4). The political friction distorts not only the present value of not defaulting but also the default value. As countries could regain access to financial markets in the future ($0 < \phi < 1$), and potentially only by future governments, the incumbent could heavily discount this benefit. This will have important implications in Section 3.5 where the possibility of under-default appears.

Definition 2 (Markov Equilibrium). An equilibrium consists of a collection of decision functions $\{g^*(\theta, b), \delta^A(\theta, b)\}$ and value functions $\{w(\theta, b), w^n(\theta, b), w^d(\theta), v(\theta, b), v^n(\theta, b), v^d(\theta)\}$, such that, given the interest rate (1), equations (3) to (7) are satisfied for all $\theta \in \Theta, b \in \mathbb{R}$.

Alternative interpretations. Although we have focused the discussion interpreting λ as political turnover and β as political polarization, this framework lends itself to multiple interpretations. Our main interpretation is based on the seminal paper by [Alesina and Tabellini \(1990\)](#). Suppose that the incumbent selects the attributes of a public good that forms the basis of total consumption. If parties disagree (are polarized) on the desirable attributes of the spending good, the utility stemming from a given level of spending will be greater for the party in power. In this case, β would capture the loss in utility due to the suboptimal allocation supplied by an alternative government. One can also think about a political environment with legislative bargaining where members of the governing coalition have access to “pork,” while those not in the coalition do not, as in [Battaglini and Coate \(2008\)](#). If q is the probability that the current legislators in power remain in the governing coalition, after a change of government, a current legislator receives pork only with probability q . Under this interpretation, we could set $\beta = q$.⁸

We could also appeal to the extensive literature on quasi-hyperbolic discounting. In general, following [Strotz \(1955\)](#) and [Laibson \(1997\)](#), it is customary to assume that individuals, besides the standard geometric discounting, placed an additional discount factor, β , between today and all future periods. The only difference from that framework is that we do so in continuous time and the additional discounting is placed randomly rather than deterministically, similar to the approach by [Cao and Werning \(2016\)](#). Indeed, it is straightforward to show that when $\lambda \rightarrow \infty$ our model maps to a continuous-time equivalent of the instantaneous quasi-hyperbolic discounting framework in [Harris and Laibson \(2013\)](#) with random taste shocks. From this point of view, the results of this paper can be interpreted as optimal regulation of individuals borrowing decisions when default is possible.

Definition of discretion. In the next section, we endow the planner with the possibility of choosing allocations other than the one implied by the rules-free equilibrium. Bear in mind some optimality conditions that shape our terminology. The first-order condition with respect to spending in equation (3) generates:⁹

$$\theta u'(g^*(\theta, b)) = w_b(\theta, b). \tag{8}$$

⁸Another interpretation related to political economy is that the preferences arise naturally from the aggregation of time-consistent preferences with heterogeneous discount rates. See [Jackson and Yariv \(2014\)](#).

⁹The fact that time is continuous allows us to use the first-order condition overcoming the difficulties present in [Krusell and Smith \(2003\)](#), as pointed out by [Chatterjee and Eyigungor \(2016\)](#). See [Cao and Werning \(2016\)](#) for similar arguments.

While the default decision is characterized by equation (5) or alternatively by the unique default threshold $b^A(\theta)$, satisfying:

$$w^n(\theta, b^A(\theta)) = w^d(\theta). \quad (9)$$

In what follows, whenever the planner chooses to respect equation (8), we say that it is endowing the governments with *discretion to spend*; while whenever the planner respects equation (5), or equivalently equation (9), it is allowing *discretion to default*.

Before analyzing the efficiency of the equilibrium, it is instructive to roughly characterize the equilibrium. Some patterns emerge that are instrumental for understanding the results in the subsequent section. In the next three lemmas, we show how the spending pattern varies with the parameters and the corresponding default pattern, including the possibility of an endogenous borrowing limit.

Lemma 1 (Spending pattern). *Suppose $u(g) = (g^{1-\gamma} - 1)/(1 - \gamma)$. When not in default, the spending growth rate is given by:*

$$\frac{\dot{g}(\theta, b)}{g(\theta, b)} = \frac{1}{\gamma} \left(\frac{\partial(r(b)b)}{\partial b} - \rho - \lambda + \lambda\beta \frac{\mathbb{E}[v_b(\theta', b)]}{w_b^n(\theta, b)} \right). \quad (10)$$

- i) For savers, as they accumulate assets $\dot{b}(\theta, b) > 0$, their spending grows $\dot{g}(\theta, b) > 0$.
- ii) For borrowers, as they accumulate debt $\dot{b}(\theta, b) < 0$, their spending declines $\dot{g}(\theta, b) < 0$.

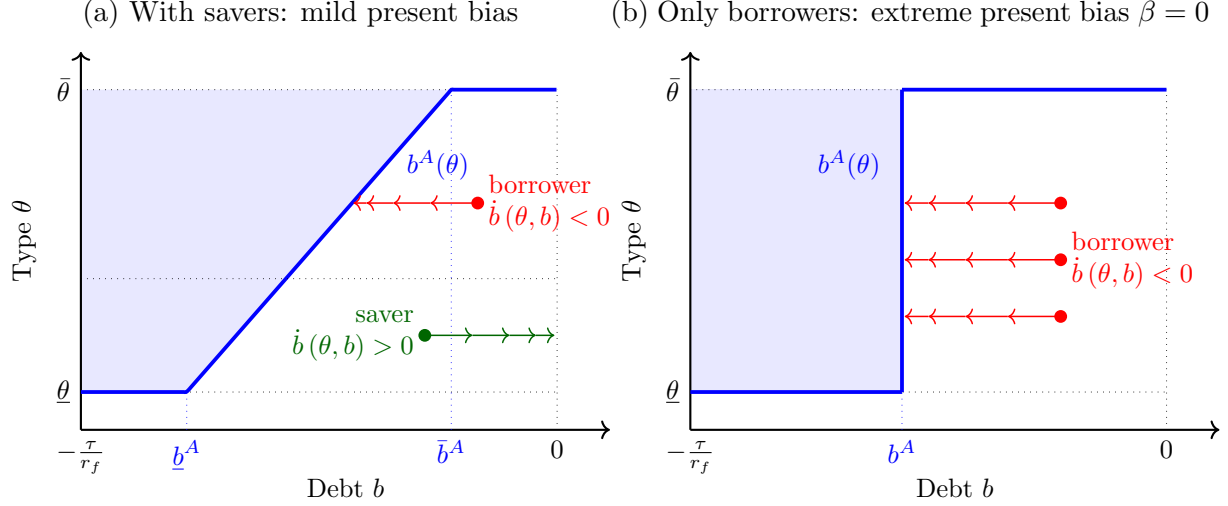
Proof: See Appendix A.2.

The key component determining the impact of the friction is $\lambda\beta \frac{\mathbb{E}[v_b(\theta', b)]}{w_b^n(\theta, b)}$. Since both $v_b(\cdot)$ and $w_b(\cdot)$ are positive, this term is also positive. It captures the incumbent's precautionary savings motive. The ratio $\frac{\mathbb{E}[v_b(\theta', b)]}{w_b^n(\theta, b)}$ determines the value of future shocks relative to the current spending needs. The larger the relative value of current wealth, $w_b(\cdot)$, the lower the spending growth. The factor $\lambda\beta$ is the arrival rate of future shocks multiplied by the future valuation; the larger the β , the more the savings.

There are two main differences with respect to what a planner would do. First, there is a direct effect because the planner, who does not penalize future governments, would have $\beta = 1$. Second, there is also a dynamic indirect effect because the planner would value the future with $v_b(\cdot) > w_b(\cdot)$. The optimal fiscal rules must deal with these two distortions to correct spending decisions.

Since the marginal value of assets $w_b(\theta, b)$ is increasing in type θ , equation (10) suggests that the growth rate of spending is decreasing in θ . The borrowing types (high θ) accu-

Figure 1: Spending and default pattern



mulate debt and decrease their spending over time. In contrast, the saving types (low θ) disaccumulate debt and increase their spending over time.

To understand the default incentive, we specify an upper bound of debt, at which all types prefer to default, and a lower bound of debt, at which no type prefers to default:

$$\underline{b}^A \equiv \inf_{\theta \in \Theta} b^A(\theta) \quad \text{and} \quad \bar{b}^A \equiv \sup_{\theta \in \Theta} b^A(\theta).$$

Lemma 2 (Default pattern). *When the financial exclusion is permanent, i.e., $\phi = 0$:*

i) The discretionary default threshold $b^A(\theta)$ is monotonously increasing in the spending needs θ : $\frac{\partial b^A(\theta)}{\partial \theta} \geq 0, \forall \theta$.

ii) If there exists savers, i.e., $\dot{b}(\underline{\theta}, b^A(\underline{\theta})) > 0$, the default threshold is strictly increasing: $\frac{\partial b^A(\theta)}{\partial \theta} > 0, \forall \theta$.

Proof: See Appendix A.3.

Lemma 2 states that we should in general expect that higher θ types are more prone to default. This is intuitive, since governments with higher spending needs are more likely to default when they have to spend part of their resources to cover debt services. As intuitive as this may appear, this is not always true. We can only prove it when the punishment for default is permanent exclusion from the financial markets. In Section 5, we provide numerical examples that the assumption that $\phi = 0$ is not innocuous.

The previous lemma implies the existence of a nondegenerate area where there is a positive default risk. However, this is not always the case; in some situations, default risk

does not emerge on the equilibrium path. The spending behavior characterized in Lemma 1 helps to characterize such situations, by distinguishing whether there are saving types or only borrowing types, as illustrated in Figure 1. Panel (a) shows an economy with mild present bias such that types with low spending needs have incentive to save for the future. In turn, Lemma 2 implies that the types separate into different default thresholds. Intuitively, if there are savers, some one could pay back the debt, and the market is willing to take a risk. In contrast, panel (b) shows an economy subject to extreme present bias $\beta = 0$, in which no types want to save for the future. An endogenous borrowing limit emerges in this situation.

The same also happens when the potential default risk premium is so high that it renders the debt burden too heavy for all government types. Since default only occurs in the event of a political turnover, the high risk premium is attributed to a high turnover λ . In this situation, the risk premium is so high such that, even if the government were to endure zero spending regardless of its spending needs, it could not afford to cover the interest payment. As an illustration, consider a debt level $\frac{1}{r_f}(1 - \kappa)\tau$. Given an interest rate of $r_f + \lambda$, the interest expense exceeds the tax revenue, $\frac{1}{r_f}(1 - \kappa)\tau(r_f + \lambda) \geq \tau$. As a result, the market imposes a borrowing limit. The default risk is zero until that limit is reached, then jumps to infinity after that. To be precise:

Lemma 3 (Endogenous borrowing limit). *If there is extreme present bias $\beta = 0$ or high political turnover $\lambda \geq \bar{\lambda}$, the market endogenously imposes a borrowing limit, $b^A(\theta) = b^A$. Moreover, depending on the cost of default:*

- i) (Permanent exclusion). If $\phi = 0$, the borrowing limit is $b^A = \frac{1}{r_f}(\kappa - 1)\tau$. The turnover threshold can be characterized as $\bar{\lambda} = \frac{\kappa}{1 - \kappa}r_f$.*
- ii) (No default cost). If there is no revenue loss $\kappa = 1$ or instantaneous reaccess $\phi = \infty$, the borrowing limit is $b^A = 0$.*
- iii) (Otherwise). If $\kappa < 1$ and $0 < \phi < \infty$, the borrowing limit is $b^A > \frac{1}{r_f}(\kappa - 1)\tau$.*

Proof: See Appendix A.4.

This is informative because even though there is no default risk, a rule-writer may have incentives to intervene. This borrowing limit may not be optimal from the point of the planner. The rule-writer may choose to make the limit tighter, defaulting before the government would, or looser, committing not to default when the government would. We analyze this possibility in Proposition 3.

3 Constrained Efficiency

In this section, we develop our main results. We start by showing that the constrained efficient allocation is simply characterized by fiscal and default rules that constrain government's actions, allowing for discretion when these rules do not apply. Then, we theoretically characterize the rules for some combinations of parameters, leaving for Section 5 the general cases.

3.1 The Mechanism-Design Problem

We study a planner, or rule-writer, who maximizes the ex ante social welfare, i.e., before the information about types is revealed, for each financial position. Referring to the system of equations (3)-(4), instead of leaving the governments to choose spending and default, the government chooses $\mathcal{M}(b) = \{g(\theta, b), \delta(\theta, b)\}$ to maximize $\mathbb{E}[v(\theta, b)]$ at every debt level b . Thus, the planner is unaffected by the political distortion β : it weights all current and future governments decisions equally.

If the planner could observe θ , only the analogous equations to (4) and (7) would be necessary. The planner would choose the optimal allocations and force governments to implement them. However, since we assume that θ is either not observable or not contractible, the planner must induce the governments to truthfully reveal their realized θ . Thus, we set up the problem as a principal-agent problem: the principal chooses the allocation contingent on $\{\theta, b\}$, restricted to truthful revelation. Hence, there are no restrictions on the set of instruments that can be used: they arise endogenously from the problem's solution.

Implementing this approach requires some modifications to (3)-(4). When the planner proposes the allocation $\mathcal{M}(b)$, any government can lie and report that it has observed an alternative value $\tilde{\theta}$, in which case the government obtains a value $w(\tilde{\theta}, \theta, b)$ given by

$$w(\tilde{\theta}, \theta, b) = (1 - \delta(\tilde{\theta}, b)) w^n(\tilde{\theta}, \theta, b) + \delta(\tilde{\theta}, b) w^d(\tilde{\theta}, \theta),$$

where the nondefault and default value functions are slight modifications of (3)-(4) satisfying

$$\rho w^n(\tilde{\theta}, \theta, b) = \theta u(g(\tilde{\theta}, b)) + \dot{b}(\tilde{\theta}, b) w_b^n(\tilde{\theta}, \theta, b) + \lambda (\beta \mathbb{E}[v(\theta', b)] - w^n(\tilde{\theta}, \theta, b)) \quad (11)$$

$$\rho w^d(\tilde{\theta}, \theta) = \theta u(\kappa\tau) + \phi (w^n(\tilde{\theta}, \theta, 0) - w^d(\tilde{\theta}, \theta)) + \lambda (\beta \mathbb{E}[v^d(\theta')] - w^d(\tilde{\theta}, \theta)). \quad (12)$$

The main difference between (3)-(6) and (11)-(12) is that the latter are evaluated at the planner's proposed allocations rather than the optimal choice of each government; and that each government can misrepresent its type to an alternative $\tilde{\theta}$. Hence the additional first

argument in the value functions. At the solution, each agent takes the truthful reporting of future governments as given. For that reason, the HJBs for v , v^n and v^d remain unaltered with respect to (4)-(7), with the exception that they are now evaluated at $\mathcal{M}(b)$ rather than the government's optimal discretionary choices. To be precise, the value functions are conditional on the mechanism, $w(\tilde{\theta}, \theta, b; \mathcal{M})$, $v(\theta, b; \mathcal{M})$. For convenience, we drop the notation \mathcal{M} , but the reader should bear in mind that the values are determined by the allocation choice.

As a result, for each a , the planner chooses $\mathcal{M}(b) = \{g(\theta, b), \delta(\theta, b)\}$ to maximize the expected social value:

$$\max_{\mathcal{M}(b)} \int_{\underline{\theta}}^{\bar{\theta}} v(\theta, b; \mathcal{M}) dF(\theta) \quad (13)$$

subject to

$$w(\theta, \theta, b; \mathcal{M}) \geq w(\tilde{\theta}, \theta, b; \mathcal{M}), \forall \theta, \tilde{\theta} \in \Theta, \forall b \quad (14)$$

$r(b)$ is given by equation (1).

Equation (14) is the truth-telling condition: a type θ prefers to tell its true type rather than imitating any other type. For convenience, after imposing truth-telling, we adopt the notation $w(\theta, b) = w(\theta, \theta, b)$, $w^n(\theta, b) = w^n(\theta, \theta, b)$, and $w^d(\theta) = w^d(\theta, \theta)$.

Remarks on the welfare function. Several implications of problem (13) are worth mentioning. First, since the planner is choosing allocations before the realization of θ , it is implicitly allowed to transfer utility across types. It can do this by manipulating the interest rate. This is especially important because of the market incompleteness. When a type θ defaults, it does not consider that its actions affect the interest rate, which other types must pay at the same level of debt. Thus, even when $\beta = 1$, the planner would like to alter the individual decisions. We discuss this effect in detail in Lemma 5.

Second, the planner chooses a mechanism that is optimal at each b , thus it does not have incentives to change it ex post. In other words, *the mechanism is sustainable*. One could consider different solution strategies. A planner could choose, with commitment, contingent spending and default future paths given an initial debt position b_0 . By committing to future potential nonoptimal allocations, the planner could manipulate future interest rates in ways that were beneficial from the initial perspective. This raises many questions about the sustainability of such policies that could render them impractical. Moreover, the impact of such a price-manipulation strategy could be minimal depending on the utility function. For instance, with a logarithmic utility function, the income and substitution of interest-rate

changes cancel out, leaving allocations unaffected.

Finally, the proposed contract is history-independent. This is likely to be irrelevant due to the *i.i.d* nature of the shocks and the fact that the budget constraint must hold every period. Allowing for monetary transfers across types could add an important role to history even when the shocks are *i.i.d*.

3.2 The Optimality of Fiscal and Default Rules

In this section, we characterize the optimal allocations. We start by characterizing the set of incentive-compatible allocations and show that they could be implemented with the combination of fiscal and default rules. We then provide some sharper results about the optimal rules. Throughout the rest of the paper, we maintain the following two assumptions:

Assumption 1. $f(\theta)$ is differentiable and satisfies

$$\frac{\theta f'(\theta)}{f(\theta)} \geq -\frac{2-\beta}{1-\beta}; \quad \forall \theta.$$

This assumption is the same as in [Amador et al. \(2006\)](#). It is a *sufficient* condition ensuring that the threshold defined in Proposition 1, equation (15), is unique. A quick inspection of the condition reveals that the left-hand side is akin to the elasticity of the density function as θ increases, which must be bounded below. We later discuss potential implications of its failure, one of which would be the possibility of “money burning” on the equilibrium path.¹⁰

Assumption 2. $g(\theta, a)$ is differentiable almost everywhere.

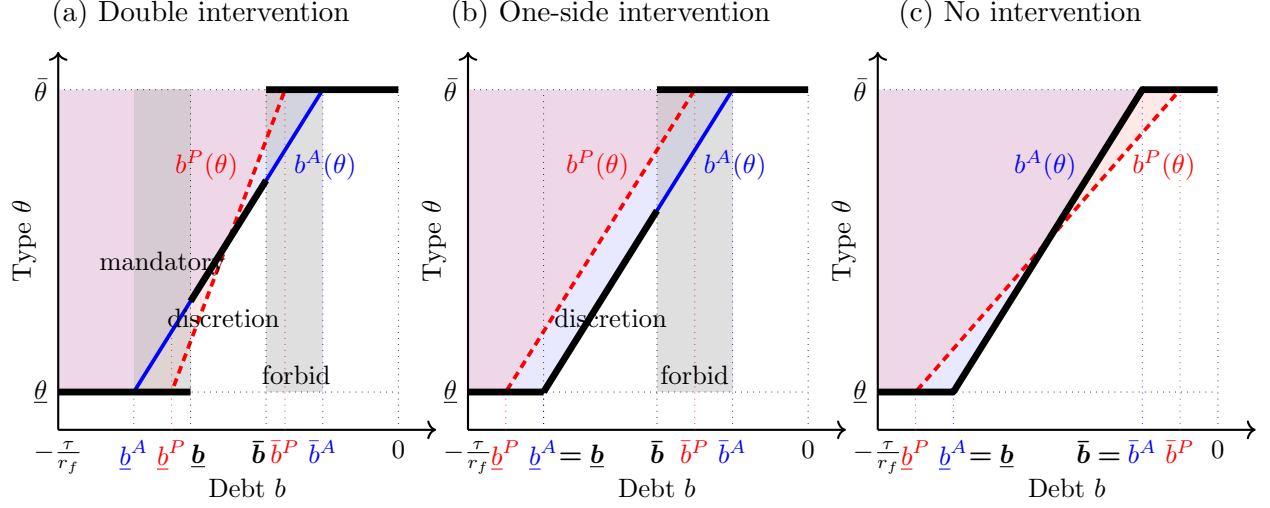
First and foremost, Assumption 2 is *necessary* to make sure that all the differential equations are well-defined. Moreover, we use the first-order approach to characterize the problem.

3.3 Incentive-Compatible Allocations

Armed with Assumptions 1 and 2, we can provide the key proposition of this paper. This result sharply characterizes the set of incentive-compatible allocations to three thresholds delimiting areas where discretion must be maintained and other areas where discretion is banned. When analyzing the implementation, we then show how these thresholds map in a straightforward fashion into fiscal and default rules.

¹⁰See [Ambrus and Egorov \(2013\)](#) for the precise conditions under which money burning could arise.

Figure 2: Default rules: why forbid or force default?



Proposition 1 (Incentive-compatible allocation). *There exist an upper bound of debt \underline{b} and a lower bound of debt \bar{b} , with $-\frac{\tau}{r_f} < \underline{b} \leq \bar{b} \leq 0$, and a debt-dependent threshold $\theta^s(b) \in \Theta$, $\forall b \geq \underline{b}$, such that*

- i) The spending rule $g(\theta, b)$ allows for discretion below the spending threshold and imposes rules above:*

$$g(\theta, b) = \begin{cases} u'^{-1}(\frac{1}{\theta} w_b^n(\theta, b)), & \text{for } \theta \leq \theta^s(b), \forall b \geq \underline{b} \\ g(\theta^s(b), b), & \text{for } \theta \geq \theta^s(b), \forall b \geq \underline{b}; \end{cases} \quad (15)$$

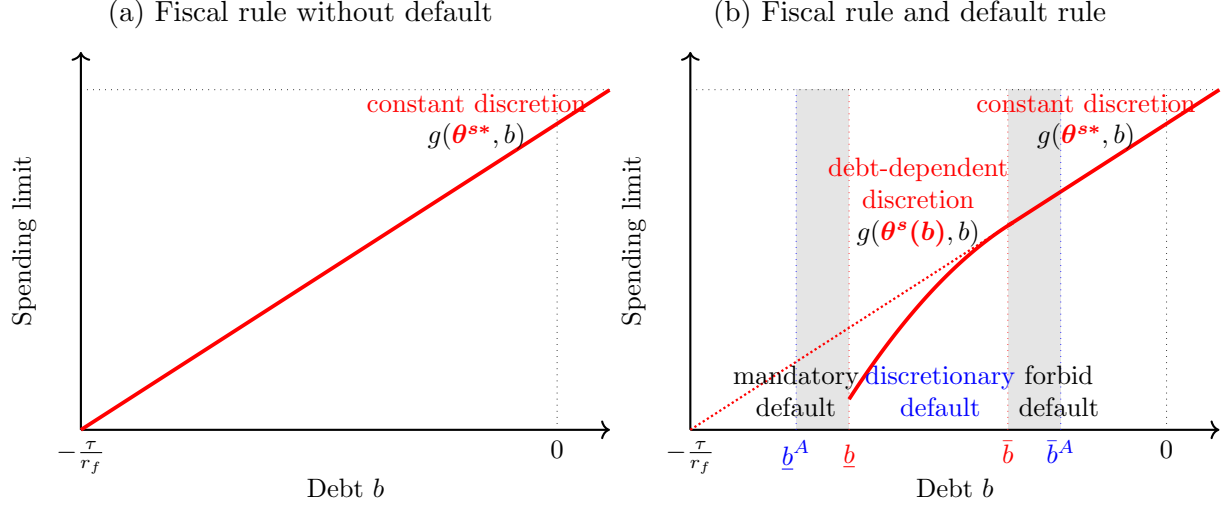
- ii) The default rule imposes mandatory default on high debt levels and forbids default for low debt levels, allowing for discretionary default for intermediate debt levels:*

$$\delta(\theta, b) = \begin{cases} 1, & \text{for } b < \underline{b}, \forall \theta \\ \delta^A(\theta, b), & \text{for } b \in [\underline{b}, \bar{b}), \forall \theta \\ 0, & \text{for } b \geq \bar{b}, \forall \theta. \end{cases} \quad (16)$$

Proof: See Appendix A.5.

Proposition 1 is simple and powerful. It states that all that the planner can do to improve outcomes is to determine the areas in which the governments are free to choose their preferred policies, while in the remaining areas they must abide by the imposed rule. Item *i)* states the space of θ governments can be split into two well-defined areas. Note that the first line of equation (15) resembles equation (8). Thus, incumbents claiming sufficiently low spending

Figure 3: Implementation



needs are endowed with discretion: they can optimally choose the desired level of spending and debt. Instead, if the incumbent claims larger than allowed spending needs, it is bound to spend no more than a predetermined amount, as shown in the second line of (15). This is true regardless of whether they are in the default area. Of course, the threshold depends on the debt level, hence the planner could tighten or loosen the allowed degree of discretion as debt is piling up. We analyze these possibilities in Section 3.4, but a priori everything is possible.

Item *ii*) states that, similarly, the space of debt can be split into regions where default is restricted and another where governments are free to choose. At debt levels where some types default and others do not, i.e., $b \in (\underline{b}, \bar{b})$, the only possible default rule that can induce truth-telling is one that allows for discretionary default, $\delta(\theta, b) = \delta^A(\theta, b)$. This is because the binary nature of the default choice makes it impossible for the principal to alter the agent's default behavior. The types that can obtain a higher value defaulting would pretend to be a non-default type, and vice versa. Thus, the intervention can only be forbidding default or mandating default. Now, the space is divided into three well-defined areas. If the debt level is neither too low nor too high, governments can discretionally decide whether to default, and they do so by following the rule in equation (5). Again, this is true independently of the spending limit in place, which changes the value functions but not the nature of the decision.

Discussion about implementation. The allocations in Proposition 1 have two components that deserve a discussion about its implementation. The first one is straightforward. To implement $\theta^s(b)$, there are several alternatives often found in the observed fiscal rules.

A debt-contingent spending cap or a debt-contingent deficit limit will easily do it. In what follows, we will call this component the *fiscal rule*. Keeping Section 2.2 in mind, adding the constraint $g \leq g(\theta^s(b), b)$, $\forall \theta, b$ to the maximization involved in equation (3) would achieve the desired outcome.¹¹ Low types would be unconstrained, choosing their preferred spending, while high θ types would meet the constraint and would only spend $g(\theta^s(b), b)$.

The implementation of the second component raises more questions. Taken literally, it implies that either the constitution or some special law, requiring a supermajority to be overturned, forbids default when the debt level is below \bar{b} , or forces it when $b \leq \underline{b}$. These kinds of rules are rare, if not completely absent, in the currently observed set of “fiscal rules.” Their unusual existence should not be a deterrent to future implementation. Moreover, the rule-writer could use indirect mechanisms without explicitly stating forbidden or mandatory default. For instance, a fiscal rule could mandate that whenever the debt level is below, say, 25% of GDP, the payments of debt services should have absolute priority in the budget. Once the threshold is exceeded, the incumbent could freely reallocate spending, including the possibility of not paying the debt obligations. This feature could be interpreted as a *relaxation* of fiscal rules when the sovereign enters the default risk region.

To impose the upper bound on debt \underline{b} , this special law could state that any debt level above a threshold, say 125% of GDP, would not be recognized as a legitimate obligation, rendering it outright illegal. Under this circumstance, the lenders would not be willing to extend additional funding, creating a de facto hard borrowing limit. For debt levels between 25% and 125% of GDP, the government would be allowed to borrow and freely default when necessary, subject to the risk premium imposed by the financial markets.

Proposition 1 also implies *the optimal convergency path* when the debt happens to be, for any reason, outside the “desired” range. As an example, the European Fiscal Compact states that the debt-to-GDP ratio of the member countries cannot exceed 60% of GDP and that the deficit should be no more than 3% of GDP. When a country exceeds the 60% threshold, a “Debt-Break-Rule” is triggered, which essentially tightens the deficit limit to at least a 1% surplus. Since many European countries are currently well above the 60% mark, the agreement has triggered a growing literature that, taking the target as given, studies the optimal debt path toward it. We believe this literature is faulty by conception or partial at best. It is not possible to study the optimal convergency path without incorporating into the framework the reason that gave rise to the threshold. They go hand in hand.

Finally, from the perspective of our theory, the European Fiscal Compact is “incomplete” in the sense that it does not provide guidelines regarding the course of action when facing default decisions. Is it implicit in the rule that governments with a debt ratio below 60% of

¹¹The same fiscal rule can be implemented with a deficit limit imposing $\dot{b} \geq \dot{b}(\theta^s(b), b)$, for all θ and b .

GDP cannot default? How far above the 60% mark is tolerable? Our theory dictates that at a certain level of indebtedness countries should be forced to default and, thus, converge instantaneously to a sound financial position.

3.4 Fiscal (Spending) Rules

In Proposition 1, we argue that $\theta^s(b)$ could be debt-dependent. There are only two elements that create this dependency. One is the presence of default risk: the planner may want to set rules that manipulate the interest rate in the right direction. The second element is the possibility of affecting the government's default decisions: different deficit/spending limits could change the government's incentives to default.

To clarify this point, it is useful to consider a benchmark economy when there is no default risk, either by taking away the possibility of default or by making the default cost so high, i.e., $\kappa = 0$ and $u(0) \rightarrow -\infty$, that no government would ever find it appealing to default for any debt level. Indeed, the government can now borrow up to the natural borrowing limit. The following lemma characterizes the optimal allocation, which specifies a constant level of discretion regardless of how much debt the government has accumulated.

Lemma 4 (No default). *When default is not possible, the lower bound of debt is the natural borrowing limit, $\underline{b} = -\frac{\tau}{r_f}$. There exists a unique spending threshold, θ^{s*} , independent of debt, characterized by:*

$$\theta^{s*} = \beta \mathbb{E}[\theta | \theta \geq \theta^{s*}]. \quad (17)$$

Proof: See Appendix A.6.

The threshold solving equation (17) is identical to the one by Amador et al. (2006). Under Assumption 1, θ^{s*} is increasing in the present-bias parameter β : the less present-biased the governments are, the more discretion is allowed. In one extreme, when there is no present bias, i.e., $\beta = 1$, the planner allows full discretion to spend: $\theta^{s*} = \bar{\theta}$. In the other extreme, with a severe present bias, i.e., $\beta \leq \underline{\theta}$, the planner bans all discretion: $\theta^{s*} = \underline{\theta}$.

It may appear puzzling that equation (17) does not involve λ . After all, political turnover is what generates the need for rules. Larger values of λ increase the frequency at which the political friction β impacts the economy, calling for stricter rules, but also increases the frequency of the genuine spending shocks, requiring better insurance and thus looser rules. All in all, both effects cancel out, rendering the threshold independent of λ .

When we allow for default risk, the characterization of the optimal threshold becomes cumbersome, not allowing for a closed-form solution. Nevertheless, we are able to implicitly characterize it:

Proposition 2 (Spending threshold). *When the financial exclusion is permanent, i.e., $\phi = 0$, the optimal spending rule satisfies: $\forall b \geq \underline{b}$,*

$$\begin{aligned}
\theta^s(b) = & \beta \mathbb{E} [\theta | \theta^s \leq \theta \leq \theta^d] + \chi(b) \left\{ \underbrace{\int_{\underline{\theta}}^{\theta^s} [w_b^n(\theta, b) - v_b^n(\theta, b)] \frac{\partial g(\theta, b)}{\partial r(b)} dF(\theta)}_{\text{Discipline effect} \geq 0} \right. \\
& + \underbrace{b \mathbb{E} [v_b^n(\theta, b)]}_{\text{Income effect} \leq 0} + \underbrace{\int_{\underline{\theta}}^{\theta^s} [g(\theta^s, b) - g(\theta, b)] \frac{\partial v_b^n(\theta, b)}{\partial r(b)} dF(\theta)}_{\text{Insurance effect} \geq 0} \\
& \left. - \underbrace{\left[\frac{1}{\lambda} \theta^d (u(g(\theta^s, b)) - u(\kappa y)) + \mathbb{E}[v(\theta', b)] - \mathbb{E}[v^d(\theta', b)] \right]}_{\text{Default manipulation} \leq 0} \right\} \frac{\partial r(b)}{\partial \theta^s}, \quad (18)
\end{aligned}$$

where the default threshold type θ^d satisfies $b^A(\theta^d) = b$.

Proof: See Appendix A.7.

There are five components determining the $\theta^s(b)$. The first component is akin to equation (17), but now the conditional expectation is over the nondefaulters, while in (17) it is unrestricted. Absent the other effects, this would generate a threshold smaller than θ^{s*} . Since the planner knows that only less tempted types remain among the nondefaulters, it can tighten the spending limit without much loss in efficiency.

The second and third components are similar to those in Halac and Yared (2018). The disciplining effect tends to increase $\theta^s(b)$. Because $\frac{\partial g}{\partial r} < 0$, an increase in the interest rate reduces the overspending, which is welfare-improving as long as $w_b^n < v_b^n$. The next component is what we term the income effect, while Halac and Yared (2018), correctly, call it the redistribution effect. In their environment, the interest rate affects outcomes through an asset-market clearing condition; thus, changes in the interest rate have redistributive effects across types. In our environment, instead, changes in the risk premium increase the debt burden for the country as a whole, which must transfer more resources to the lenders. Also, as in their paper, the net contribution of these two effects is ambiguous, depending on the value of β . However, in their environment, since bond is in constant (zero) net supply, there is no ‘‘aggregate’’ debt effect. In our case, instead, as the debt level increases, the negative impact tends to tighten the spending limit by reducing the discretion region.

The last two components do not have an equivalence in literature. What we call the *insurance effect* arises because of the change in the planner’s marginal value of wealth,

which is absent in Halac and Yared (2018) since they focus on a two-period economy. This provides an additional incentive for the planner to increase $\theta^s(b)$, with a positive impact on the interest rate. The larger interest rate allows all types with discretion to obtain a larger continuation value.

The final component is the default manipulation. Unfortunately, we are not able to sign this component. This happens because, as we show in the next section, it is not clear whether the economy experiences excessive or insufficient default. Both cases are possible, which could lead the planner to either relax or tighten the constraint. We analyze this effect quantitatively in Section 5.

It is clear from Proposition 2 that these additional effects depend on the debt level and appear only when there is default risk. Thus, for debt levels for which the risk of default is zero, the optimal fiscal rule resembles that in equation (17). In addition, for the fiscal rule to be meaningful, it must apply to governments that actually have a spending choice. Let $\theta^d(b)$ be the type's default threshold for each b . In Section 3.5, we provide conditions under which it is unique and monotone increasing, in which case all types $\theta \geq \theta^d(b)$ default. It is evident that $\theta^s(b) > \theta^d(b)$ would render the fiscal rule innocuous, since all constrained governments would default. Moreover, when the present bias is strong enough, the direct effect due to (17) dominates, then we have:

Corollary 1 (Debt dependency). *For any combination of parameters:*

- i) (Severe present bias). If $\beta \leq \underline{\theta}$, there is no discretion to spend $\theta^s(b) = \underline{\theta}$, $\forall b \geq \underline{b}$.*
- ii) (Mild present bias). If $\beta > \underline{\theta}$, there is discretion. In the nondefault area, the threshold is constant. In the default risk area, there could be increased or reduced discretion.*

$$\theta^s(b) \begin{cases} = \theta^{s*}, & \text{for } b \geq \bar{b} \\ \leq \theta^d(b), & \text{for } b \in [\underline{b}, \bar{b}). \end{cases}$$

Loosely speaking, in Corollary 1, part ii) implies that as the debt level rises there is a race between the solution to the threshold in Proposition 2 and the default threshold. If the income effect and the default manipulation effect are not powerful enough, it may be optimal to lift the fiscal limits and allow the governments to discretionarily default, if they have large spending needs, or to freely choose spending, if their needs are moderate or small. On the equilibrium path, this could be implemented with either unaltered fiscal rules as the debt level increases (but ineffective) or complete elimination of all fiscal rules.

3.5 Default Rules

Before characterizing the constrained efficient default policies, it is informative to analyze what the planner would do if she had perfect information about θ . From now on, we denote by $\delta^P(\theta, b)$, the optimal default policy under a *perfect-information* benchmark.

Lemma 5 (Unconstrained optimal default). *If the principal had perfect information, it would choose to default $\delta^P(\theta, b) = 1$ if and only if*

$$v^n(\theta, b) + \lambda \mathbb{E} \left[\frac{\partial v^n(\theta', b)}{\partial r(b)} (1 - \delta^P(\theta', b)) \right] \leq v^d(\theta). \quad (19)$$

Correspondingly, there is a desired default threshold for the principal $b^P(\theta)$ at which equation (19) holds with equality.

Proof: See Appendix A.8.

Equation (19) makes clear that even when the planner has perfect information, it chooses $\delta^P(\theta, b) \neq \delta^A(\theta, b)$. To see this, note that when $\beta = 1$, $v^n(\theta, b) = w^n(\theta, b)$. Since a government would default when $w^d(\theta, b) \geq w^n(\theta, b)$, it implies $v^d(\theta, b) \geq v^n(\theta, b)$. This happens because the principal takes into consideration the interest-rate effect for all possible θ 's. If one type defaults, it increases the interest rate that lenders charge to all types. In this sense, the implications for default rules are different than for spending rules. Recall that in Lemma 4, we show that the equilibrium converges to the first best as $\beta \rightarrow 1$, so that fiscal rules are not needed. However, when default is possible, the equilibrium may be inefficient even when there is no present bias. While the principal does not need to impose spending limits, its default incentive differs from that of the agent.

We now turn to the optimal intervention. We start by showing that even when there is no default risk, as long as $\beta < 1$, the planner would like to alter the default decisions. When analyzing the rules-free equilibrium, we show, in Lemma 3, that one possible outcome is a market-imposed endogenous borrowing limit. One implication of it is that there is no default risk on the equilibrium path. Still, the borrowing limit imposed by the market may not be optimal from the point of view of the planner. Now we show how the fiscal rules can also generate an endogenous borrowing constraint. When this happens, we say that the government takes away all the discretion to default and imposes a *debt limit rule*. The rule states that a government can borrow only up to the limit and never default. If the government happens to start with a debt level beyond the limit, then it must default. In parallel to Lemma 3, the following proposition describes conditions under which a debt limit is optimal.

Proposition 3 (Optimal debt limit). *If the present bias is severe $\beta \leq \underline{\theta}$ or the risk premium is high $\lambda \geq \bar{\lambda}$, the optimal default rule is a debt limit b^* . Moreover, depending on the cost of default:*

i) (Permanent exclusion). If $\phi = 0$, the debt limit coincides with the rules-free equilibrium:

$$b^* = b^A = \frac{1}{r_f}(\kappa - 1)\tau. \quad (20)$$

ii) (No default cost). If there is no revenue loss $\kappa = 1$ or instantaneous reaccess $\phi = \infty$, the debt limit coincides with the rules-free equilibrium, $b^ = b^A = 0$.*

iii) (Otherwise). If $\kappa < 1$ and $0 < \phi < \infty$, the planner imposes a tighter limit and defaults sooner, $b^ > b^A > \frac{1}{r_f}(\kappa - 1)\tau$.*

Proof: See Appendix [A.9](#).

There are two conditions under which a debt limit is optimal, when the present bias is severe, $\beta \leq \underline{\theta}$, and when the risk premium is large, $\lambda \geq \bar{\lambda}$. Since the mechanisms are different, we discuss them separately.

When $\beta \leq \underline{\theta}$, regardless of the incumbent's spending needs, even the least-tempted type $\underline{\theta}$ is too myopic to save on behalf of future governments with higher spending needs. As a result, all types are borrowing and accumulating debt, $\dot{b}(\theta, b) \leq 0, \forall \theta$. In this situation, Lemma 4 applies. That is, the planner takes away all discretion to spend and imposes the same spending on all types. As they approach their default threshold, they stop accumulating debt and stay at the threshold. Their spending is exactly equal to their net income after interest payments, $r_f b + \tau$, which does not depend on θ . Therefore, all types have the same default threshold, $b^A = \underline{b}^A = \bar{b}^A$. Thus, due to the fiscal rule, a borrowing limit endogenously arises: the market stops lending to all types, knowing that they would default. There is no discretionary default region.

Is this market-imposed limit optimal? Recall that the discrepancy in the default incentives between the planner and the agent due to present bias manifests in two ways: the agent discounts not only the continuation value of not defaulting, but also the continuation value of defaulting. So, it is possible that the agent defaults too much too early or too little too late. In case *i*), with a permanent financial exclusion, it turns out that the default incentives between the planner and the agent are exactly aligned. Intuitively, once in default, the economy falls into permanent autarky with spending fixed to $\kappa\tau$. At the borrowing limit, the government also faces a ‘‘permanent’’ constant spending $r_f b^* + \tau$. The two must equate so that the government is indifferent between defaulting or not, which generates the borrowing

limit in equation (20). This also makes clear why the incentives are aligned. Since both are comparing constant streams of consumption, the excess discounting by the government becomes irrelevant. It has the same effect on the default and nondefault states.

It follows from the borrowing limit that when $\kappa = 1$, the market would not be willing to lend, and the borrowing limit is $b^* = 0$. The same outcome also occurs when there is immediate financial reaccess. Since the government can restart instantly with a clean slate with zero debt burden, it will default on any amount of debt. Thus, case *ii*) resembles a “rainy day fund”: the government cannot borrow but can save up for rainy days.

Whenever default is costly, $\kappa < 1$ and financial exclusion is temporary $0 < \phi < \infty$, the debt capacity is positive. In this case, perhaps counterintuitively, the agent defaults too little too late. What happens is that the benefits of default happen after the costs, so the myopic discounting has a larger effect on the default value. One of the main benefits of default arrives later: upon reentry, the government starts with no debt. A government that mostly cares about its own term overweights the immediate costs under financial exclusion and does not internalize the future benefits that another government can enjoy. The planner who weights all governments alike wants to default sooner and before the agent. As a result, the planner imposes a tighter debt limit and *forces default* whenever the government is holding on to too much debt.

When the debt limit arises because $\lambda > \bar{\lambda}$, the outcome is similar, but the underlying mechanism is different. As shown in Lemma 3, the market discipline induces an endogenous borrowing limit. The high risk premium demanded by the market makes it impossible for the late-default types to separate themselves from the early-default ones. For ease of exposition, suppose financial exclusion is permanent, i.e., $\phi = 0$. With an endogenous borrowing limit, the debt capacity would be $b^A = \frac{(\kappa-1)\tau}{r_f}$. If there is a type contemplating borrowing slightly more and delaying default, the market would demand an interest rate $r_f + \lambda$. Thus, the total interest payments would be $\frac{(1-\kappa)\tau}{r_f}(r_f + \lambda)$, which if $\lambda > \bar{\lambda}$ would generate negative spending. This implies, for instance, that in environments with instantaneous gratification as in Harris and Laibson (2013), where $\lambda \rightarrow \infty$, there cannot be default in equilibrium, and the planner should impose a tighter debt limit than the market.

The endogenous borrowing limit results are instructive about the directions on which the planner wants to move the default thresholds, but abstract from the risk-premium effect. In reality, the risk premium is prevalent and the main subject of policymakers’ concern. It only arises when neither the present bias nor the uncertainty is too large. It is cumbersome to characterize all the possibilities sharply. However, we can state the following result:

Proposition 4 (General characterization). *Suppose the present bias is mild, $\beta > \theta$, and there is low turnover $\lambda < \bar{\lambda}$. Then, depending on the spending needs heterogeneity,*

i) (Some savers). If the taste distribution is dispersed, i.e., the lowest type satisfies:

$$\underline{\theta} < \frac{\lambda\beta}{(\rho + \lambda - r_f)} \frac{\mathbb{E}[v_b(\theta, b^A)]}{u'(r_f b^A + \tau)}, \quad (21)$$

there exists θ such that $\dot{b}(\theta, b) > 0$, for each b . Default optimally happens on the equilibrium path. Moreover, when $\phi = 0$, the planner sets $\underline{b} = \underline{b}^A$ and $\bar{b} < \bar{b}^A$.

ii) (All borrowers). Otherwise, all types borrow, $\dot{b}(\theta, b) \leq 0, \forall \theta, b$. There is an endogenous borrowing constraint b^A ; hence, the optimal default rule is a debt limit b^ .*

Proof: See Appendix A.10.

The condition in equation (21) provides a necessary condition for the existence of risk premium on the equilibrium path. Unfortunately, we can only go as far as to characterize it as a function of the (endogenous) value $v_b(\theta, b)$. Nevertheless, it is useful to interpret the condition under which a government saves or borrows. When equation (21) is violated, all possible governments want to increase their debt positions, and even the less tempted would not be willing to reduce debt. As a result, the market again imposes a borrowing limit. Intuitively, lenders are willing to lend today only if someone in the future would be willing to pay back. As stated in *ii)*, the outcome then is similar to Proposition 3, except that in this case the fiscal rule could allow for some discretion to spend.

Instead, when condition (21) is satisfied, both savers and borrowers co-exist, which gives rise to the possibility of lending with potential for repudiation. There is a nondegenerate area of debt $[\underline{b}, \bar{b}]$, where sometimes default happens and others times it does not. Who are the defaulters and who are not depend on the probability of regaining access to the financial markets. When there is permanent exclusion, $\phi = 0$, in Lemma 2 we characterize the type-dependent default threshold $b^A(\theta)$ and show that it is strictly increasing in θ . In other words, the high-need types default at a lower level of debt, while the lowest-need type, $\underline{\theta}$, is the last willing to default.

Due to Proposition 1 we know that the rule-writer cannot alter the default decisions on the interior of $[\underline{b}, \bar{b}]$, it can only change the borders. But then Proposition 3, *i)* also applies to the determination of \underline{b} : if all types want to default when their debt position is below \underline{b} and $\phi = 0$, the planner does not want to distort that decision. However, at the upper bound, things are different. Only a measure zero of agents would find it optimal to default, which triggers some inefficient effects. If the upper bound were $\bar{b} = b^A(\bar{\theta})$, two inefficiencies would play an important role. First, because $\beta < 1$, the high types initially defaulting do not accurately internalize that by defaulting they are forcing other less-tempted but still

high-need types to financial autarky, imposing on them low consumption. Second, the sole possibility that this upper-bound type can default increases the risk premium to all other governments, which, as we explain after Lemma 5, is not internalized by any government, irrespective of β . Both effects point to the same policy intervention: the planners can increase welfare by imposing an upper bound $\bar{b} < b^A(\bar{\theta})$. This is what we call the forbidding default region in Proposition 1.

When $\phi > 0$, the outcome and optimal policy are difficult to characterize. The main problem arises because the default threshold may no longer be monotone in θ . This creates some problems that make a formal proof difficult. Nevertheless, in the next section, we quantitatively evaluate the optimal problem, and we show analogous results.

4 Extensions

4.1 Partial Forgiveness and Suspension of Payments

So far, we have assumed full forgiveness, so that upon reaccessing the market, previously defaulted debt is fully discharged. However, as documented by [Arellano, Mateos-Planas, and Ríos-Rull \(2019\)](#), default is often partial, with only a portion of the debt discharged. In these situations, upon reaccessing the market, the government may need to repay some of the previously defaulted debt. To address this issue, we adopt an alternative assumption. Suppose that financial exclusion is temporary $\phi > 0$ and that upon reentry the government must repay a proportion $\alpha \in [0, 1]$ of the defaulted debt. Thus, if the current government, or any future one, regains access to the market, it starts with debt level, after adjusting for the haircut, equal to: $b^h = \alpha b$.¹² When $\alpha = 1$, default leads to *suspension of payments*. When $\alpha = 0$, default leads to debt being fully discharged as in the baseline model.

Under this assumption, when not in default, the value functions are identical to those before, except that the risk premium must be modified to incorporate the recovery value. The most substantial difference arises in the computation of the default value functions. Now upon reaccessing the market the values depend on the past defaulted debt. We denote the default value functions by $w^d(\theta, b^h)$ and $v^d(\theta, b^h)$. They satisfy the following HJB equations:

$$\rho w^d(\theta, b^h) = \theta u(\kappa\tau) + \phi (w^n(\theta, b^h) - w^d(\theta, b^h)) (1 - \delta_d(\theta, b^h)) + \lambda (\beta \mathbb{E}[v^d(\theta', b^h)] - w^d(\theta, b^h)) \quad (22)$$

$$\rho v^d(\theta, b^h) = \theta u(\kappa\tau) + \phi (v^n(\theta, b^h) - v^d(\theta, b^h)) (1 - \delta_d(\theta, b^h)) + \lambda (\mathbb{E}[v^d(\theta', b^h)] - v^d(\theta, b^h)) \quad (23)$$

¹²For simplicity, we assume that when a government is in default interest payments do not accumulate.

In equations (22) and (23), the changes in value in the event of financial reaccess are multiplied by $(1 - \delta_d(\theta, b^h))$. When $\alpha = 0$, as in the baseline model, all debt is discharged, thus whenever a government has the chance to reenter the financial markets, it does it with certainty. But when $\alpha > 0$, an incumbent could disregard the opportunity and remain in default. This could be interpreted as immediate default, as prescribed by the equilibrium default decision or the default rule. In this case, reentry into the financial markets happens only when a government with sufficiently low spending needs is willing to resume the debt payments. Note that we have defined this rejection to reenter function with a subscript d , $\delta_d(\cdot)$, to stress that it may not coincide with the previous function $\delta(\cdot)$.

When the country is in a nondefault state, the default condition is analogous to the previous condition (9) now modified to:

$$w^n(\theta, b^A(\theta)) = w^d(\theta, \alpha b^A(\theta)). \quad (24)$$

Regarding the reentry condition, we assume that the haircut $(1 - \alpha)$ is not applied again whenever the government rejects the option to enter. Then, $\delta_d(\cdot)$ satisfies:

$$\delta_d(\theta, b^h) = \begin{cases} 1, & \text{if } w^n(\theta, b^h) < w^d(\theta, b^h) \\ 0, & \text{if } w^n(\theta, b^h) \geq w^d(\theta, b^h). \end{cases} \quad (25)$$

This equation points to two special cases. In the baseline model $\alpha = 0$, then because it is always true that $w^n(\theta, 0) > w^d(\theta, 0)$, we have $\delta_d(\theta, 0) = 0$ for all θ . Another special case is when there is a suspension of payments. Since in this case $\alpha = 1$ we have that $\delta_d(\theta, b) = \delta(\theta, b)$ for all θ and all b .

In addition, introducing some recovery value changes the risk premium charged on loans. Since lenders could recover some of the funds lent, the risk premium must take into account this additional benefit. To be precise, now the interest rate must satisfy:

$$r(b, b^h) = r_f + \lambda \mathbb{E} [\delta(\theta, b) (1 - R(\theta, b^h))],$$

where $R(\theta, b^h; \delta_d)$ is the per unit recovery value of a defaulted loan when a type- θ borrower is required to, conditional on reentry, pay back b^h dollars per each b borrowed. In Appendix A.11, we show that $R(\cdot)$ is linear in b^h , and therefore the risk premium per unit of loan can be written as:

$$r(b) = r_f + \lambda \mathbb{E} [\delta(\theta, b) (1 - \alpha \hat{R}(\theta, b^h))]. \quad (26)$$

This equation also makes clear that because of the linearity of the recovery value on the amount to be repaid, it can be decomposed into the haircut, $1 - \alpha$, and the present expected

value of the probability of reentry, $\hat{R}(x) = R(1, x)$. In Appendix A.11, we show that:

$$\hat{R}(\theta, b^h) = \begin{cases} \frac{\lambda\phi(1-\mathbb{E}[\delta_d(\theta, b^h)])}{\lambda\phi(1-\mathbb{E}[\delta_d(\theta, b^h)])+r_f(r_f+\lambda+\phi)} & \text{if } \delta_d(\theta, b^h) = 1. \\ \frac{r_f+\lambda}{r_f+\lambda+\phi} \frac{\lambda\phi(1-\mathbb{E}[\delta_d(\theta, b^h)])}{\lambda\phi(1-\mathbb{E}[\delta_d(\theta, b^h)])+r_f(r_f+\lambda+\phi)} & \text{otherwise.} \end{cases} \quad (27)$$

Under suspension of payments, i.e., $\alpha = 1$, the last equations simplify even further since $\delta_d(\cdot) = \delta(\cdot)$, which also ensures that the government initially defaulting would not reenter, even when it has the chance. This implies that only the first line of (27) applies. As a result, the default incentives are analogous to the ones under full debt forgiveness with permanent exclusion ($\phi = 0$). To see why, consider a type θ who defaulted the amount a , then $\delta(\theta, b) = \delta_d(\theta, b^h) = 1$. Since it must repay the same amount of debt, this type would never return to the market even if given the opportunity. Thus, the direct effect on default incentives is akin to permanent exclusion. Therefore, the results in Lemma 2 and 3 still apply: default $\delta(\theta, b)$ is monotonously increasing in type θ and the debt limit due to high risk premium, with the same $\bar{\lambda}$, follows without modifications. It then follows in a straightforward way that:

Lemma 6 (Suspension of Payments). *When $\alpha = 1$, for any $\phi > 0$, the monotone default pattern in Lemma 2 holds as in the case when $\phi = 0$. Correspondingly, the qualitative characterization of Propositions 2, 3, and 4 still hold conditional on an alternative $\phi = 0$.*

The previous lemma states that an economy with the suspension of payments can be, *at least qualitatively*, analyzed as an economy with full debt forgiveness and permanent exclusion upon default. Of course, the *quantitative results are not the same*. There are additional indirect effects through the continuation values and the level of the risk premium that would generate different quantitative rules. Lemma 6 stresses the potential substitutability between ϕ and the haircut. With the suspension of payments, this substitutability is as large as possible. However, when $\alpha < 1$ and $\phi > 0$, the substitution is not perfect, generating further quantitative implications. We leave that analysis for the quantitative part of Section 5.

4.2 Business Cycles

In this section, we relax the assumption that τ is constant, allowing for a mean-reverting tax revenue. To ensure that τ is always positive, we assume that it follows a Cox–Ingersoll–Ross process:

$$d\tau = \nu(\bar{\tau} - \tau) + \sigma_\tau\sqrt{\tau}dW, \quad (28)$$

where W is a standard Brownian motion.

We further assume that τ is perfectly observable and contractible. One may wonder about the informational asymmetry between τ and θ . The main premise here is that the availability of resources to a government can be measured, if not perfectly, almost certainly, so it is possible to write contracts contingent on it.¹³ While θ can be interpreted as the needs generated by the revenue shock. These needs even though real can arise from subjective assessments of the economic situation or could be difficult to verify. Alternatively, one can think that the business cycle is driven by a unique process that has an observable component, τ , and a nonmeasurable component and is therefore noncontractible.

The addition of this shock affects the baseline environment in two main ways. First, the allocation, the interest rate, and all value functions will now depend on the additional state τ . Thus, an allocation now will be $\{g(\theta, b, \tau), \delta(\theta, b, \tau)\}_{\theta \in \Theta, b \in \mathbb{R}, \tau \in \mathbb{R}_+}$. The interest rate is:

$$r(b, \tau) = r_f + \lambda \mathbb{E}[\delta(\theta, b, \tau)].$$

To write down the last equation, we have used the arguments of Section 2.1 to show that the interest rate is still independent of θ , because only a jump in the type can generate a default. The revenue process is a smooth one, which never triggers default directly. However, now the default rate does depend on this additional source of uncertainty, but only indirectly as it changes the regions of defaulters and nondefaulters.

Finally, the “continuously” moving shock to τ requires a modification to the value functions. Here we present only the difference with Section 2.2, but the analogous changes for the mechanism-design problem should be clear to the reader. The set of HJB equations (3)-(6) and (4)-(7) for the value functions are replaced by:

$$\begin{aligned} (\rho + \lambda)w^n(\theta, b, \tau) &= \max_g \{ \theta u(g) + (r(b, \tau)b + \tau - g)w_b^n(\theta, b, \tau) \} + \lambda \beta \mathbb{E}[v(\theta', b, \tau)] \quad (29) \\ &+ \nu(\bar{\tau} - \tau)w_\tau^n(\theta, b, \tau) + \frac{1}{2}\sigma_\tau^2 \tau w_{\tau\tau}^n(\theta, b, \tau). \end{aligned}$$

$$\begin{aligned} (\rho + \lambda)v^n(\theta, b, \tau) &= \theta u(g^*) + (r(b, \tau)b + \tau - g^*)v_b^n(\theta, b, \tau) + \lambda \mathbb{E}[v(\theta', b, \tau)] \quad (30) \\ &+ \nu(\bar{\tau} - \tau)v_\tau^n(\theta, b, \tau) + \frac{1}{2}\sigma_\tau^2 \tau v_{\tau\tau}^n(\theta, b, \tau). \end{aligned}$$

¹³This may not be true in countries with weak institutions. For instance, Argentina from 2011 to 2015 was consistently misreporting not only inflation but also GDP. Still, because tax revenue must be shared with independent provinces, the Federal government had to report the true tax revenue.

$$\begin{aligned}
(\rho + \lambda + \phi)w^d(\theta, \tau) &= \theta u(\kappa\tau) + \phi w^n(\theta, 0, \tau) + \lambda\beta\mathbb{E}[v^d(\theta', \tau)] \\
&+ \nu(\bar{\tau} - \tau)w_\tau^d(\theta, \tau) + \frac{1}{2}\sigma_\tau^2\tau w_{\tau\tau}^d(\theta, \tau).
\end{aligned} \tag{31}$$

$$\begin{aligned}
(\rho + \lambda + \phi)v^d(\theta, \tau) &= \theta u(\kappa\tau) + \phi v^n(\theta, 0, \tau) + \lambda\mathbb{E}[v^d(\theta', \tau)] \\
&+ \nu(\bar{\tau} - \tau)v_\tau^d(\theta, \tau) + \frac{1}{2}\sigma_\tau^2\tau v_{\tau\tau}^d(\theta, \tau).
\end{aligned} \tag{32}$$

The main difference with respect to Section 2.2 is the extra terms in the second line of each equation. These terms capture the effects of the shocks to τ on the value functions. They clearly affect their levels and shapes but do not change the fundamental structure of the problem and hence their optimal conditions. Following similar steps to proof of Proposition 1, it readily follows that:

Lemma 7 (Random Revenues). *If the government's revenue follows the process in equation (28), the optimal intervention has the same pattern as Proposition 1. There exist debt thresholds $\underline{b}(\tau)$ and $\bar{b}(\tau)$, with $-\frac{\tau}{r_f} < \underline{b}(\tau) \leq \bar{b}(\tau) \leq 0$, and a state-dependent threshold $\theta^s(b, \tau) \in \Theta$, $\forall b \geq \underline{b}(\tau)$, such that*

- i) All types $\theta \leq \theta^s(b, \tau)$ have discretion to spend, while those above abide by the rule.*
- ii) If $b < \underline{b}(\tau)$, all types are forced to default, while if $b > \bar{b}(\tau)$, default is banned. In between governments have discretion to default.*

Proof: Online Appendix.

Lemma 7 is an extension of Proposition 1. It shows that the main properties shown in the previous sections remain, adding a dependency of the optimal rules on the observable state of the economy. As one can see, the notational burden grows considerably by the addition of the extra state. For this reason, we avoid the characterization of the thresholds and rely on numerical results. It is interesting, though, that this section adds a requirement that the optimal fiscal rules be dependent on the current state of the economy. Regarding our discussion in Section 3.3 about implementation, this result adds the requirement that the rules should be contingent on the observable state of the economy. This dependency is, for instance, currently absent in the European Fiscal Compact.

5 Quantitative Application

In this section, we aim to understand some quantitative implications of our theory. Our goal is general, not focused on a particular country, but it is relevant to understand the

quantitative prescriptions of the theory for alternative “standard” cases. For this reason, we quantitatively analyze three cases that we named: Germany, Greece, and Italy. This naming is motivated by some broad empirical patterns, but by no means should the naming be interpreted as accurate representations of the countries. We broadly see Greece as having a relatively low debt capacity and a risk premium sensitive to debt accumulation. We see Germany as having a large debt capacity and a risk premium only mildly sensitive to debt accumulation. And we characterize Italy as having both a large debt capacity and a high risk premium sensitivity. In this spirit, we calibrate these three “countries.”

5.1 Calibration

We specify a constant relative risk aversion (CRRA) utility function for spending with risk aversion parameter γ . The spending needs follow a truncated lognormal distribution, i.e., $\theta \sim LN(-\frac{1}{2}\sigma^2, \sigma)$ in the domain $[\underline{\theta}, \bar{\theta}]$. To ensure that its expected value is normalized to 1, we set the upper bound $\bar{\theta} = 1/\underline{\theta}$. Given the functional forms, the model consists of a set of eleven parameters $\{\gamma, \rho, r_f, \tau, \lambda, \beta, \sigma, \underline{\theta}, \bar{\theta}, \kappa, \phi\}$.

Since none of these countries had fiscal rules before 1993, we calibrate the model-generated moments of the rules-free equilibrium to the pre-1993 data wherever possible. We select the data sample prior to the year the Maastricht Treaty came into force to ensure that no fiscal rules were imposed. Appendix B.1 includes details on data sources and measurements. All level variables are normalized relative to GDP. The model is calibrated at an annual frequency.

Table 1 reports the calibrated parameters and the corresponding data moments. We set the risk aversion parameter γ to 1. The risk-free rate r_f is calibrated to match an annual risk-free interest rate of 4%. The discount rate ρ is set to equal to the risk-free rate, which avoids borrowing due to any gap between the interest rate and the discount rate. The tax revenue parameter τ corresponds to the size of government revenue (percentage of GDP). For the three European countries we study, the average tax revenue is around 45%.

We calibrate the reaccess rate following empirical work on sovereign default and subsequent financial exclusion. Cruces and Trebesch (2013) document that the duration of financial exclusion depends on the extent of haircut in the event of sovereign default, with restructuring involving higher haircuts associated with longer periods of exclusion. They estimate that, in a default scenario involving a haircut of 60% or above, the probability of remaining excluded after 10 years is slightly over 50%. This estimate implies a reaccess rate of 0.03. Given our baseline model assumption of full default, we set a slightly lower reaccess rate ϕ at 0.02.

Table 1: Parameters and moments

Parameter	Symbol	Value			Moment
		Germany	Italy	Greece	
CRRA	γ		1		preset
Discount rate	ρ		0.04		interest rate
Risk-free rate	r_f		0.04		interest rate
Tax revenue	τ		0.45		government revenue (% of GDP)
<i>Spending needs</i>					
Turnover rate	λ	1/15	1/8	1/8	corr(default risk, debt level)
Present bias	β	0.75	0.75	0.5	average debt growth
Distribution std dev	σ		0.5		dispersion in debt growth
Type lower bound	$\underline{\theta}$		0.2		default risk debt upper bound
Type upper bound	$\bar{\theta}$		5		inverse of $\underline{\theta}$ to normalize mean
<i>Cost of default</i>					
Output loss	κ		0.89		default risk debt lower bound
Reaccess rate	ϕ		0.02		Cruces and Trebesch (2013)

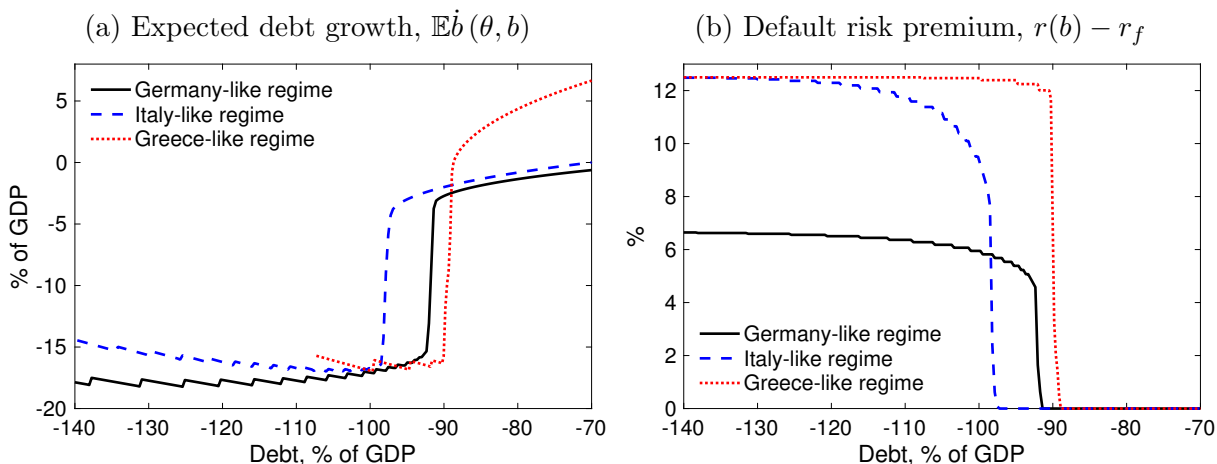
We estimate the remaining five parameters by taking advantage of the spending and default risk patterns in the rules-free equilibrium. First, regarding the spending pattern, the present-bias parameter β can be identified from the average debt growth, as it directly corresponds to the overspending tendency. Instead, the variance of taste distribution σ maps directly to the dispersion in debt growth. Second, regarding the default risk, the output loss parameter κ affects the lower bound of debt \bar{b}^A , at which default risk starts to emerge. Further, the spending-needs bounds are calibrated to ensure that the upper bound of debt \underline{b}^A is at least above the one observed in the data. Finally, the correlation between debt level and default risk premium $\text{corr}(r(b), b)$ allows us to identify the turnover rate λ .

5.2 Quantitative Regimes

To differentiate the three different regimes of interest, we rely on two parameters: political turnover and political polarization. The first regime that we study, Germany, is characterized by a low turnover and a mild present bias. The second one is the Italy-like regime, which has a high turnover and a mild present bias. The last one, Greece (or an Argentina-like regime), has a high turnover and a severe present bias. All the calibrated parameters are listed in Table 1.

While all three regimes fit into case *i*) of Proposition 4 where condition (21) is satisfied, the range of default risk area differs quantitatively. Panel (b) of Figure 4 plots the default risk premium at different levels of government debt. In all three regimes, when government

Figure 4: Equilibrium default risk area



debt exceeds around 90% of GDP, default risk starts to emerge. Both the Germany-like and the Italy-like regimes exhibit a wide range of debt levels with default risk. This is because the mild present bias leads to many saving types. However, the default risk premium still reacts very differently depending on the turnover rate: in the Italy-like regime, given that the spending shocks associated with turnover occur at a higher frequency, the default risk premium jumps up by a larger magnitude as debt accumulates beyond a certain threshold, while the Germany-like regime experiences a milder increase in default risk premium. In contrast, the Greece-like regime has a very narrow band of debt levels with default risk. This is because when present bias becomes more severe, fewer types save. In this regime, compounded by the high turnover risk, the default risk premium jumps up sharply when debt increases.¹⁴

5.3 Policy Analysis

We start by analyzing the optimal default rules in the Greece-like regime. In our classification, this is a country with high turnover and high political polarization. Thus, in the absence of any rules, it is characterized by a low debt capacity with a highly sensitive risk premium. We depict this situation with the blue dashed line in Panel (a) of Figure 5. For levels of debt above 90% of GDP, most θ -types would default, generating the almost vertical line around 90%. The few types that do not default are depicted along the positively sloped line between 90% and 110% of GDP.

The first exercise we perform is to analyzing the optimal default rules when the planner

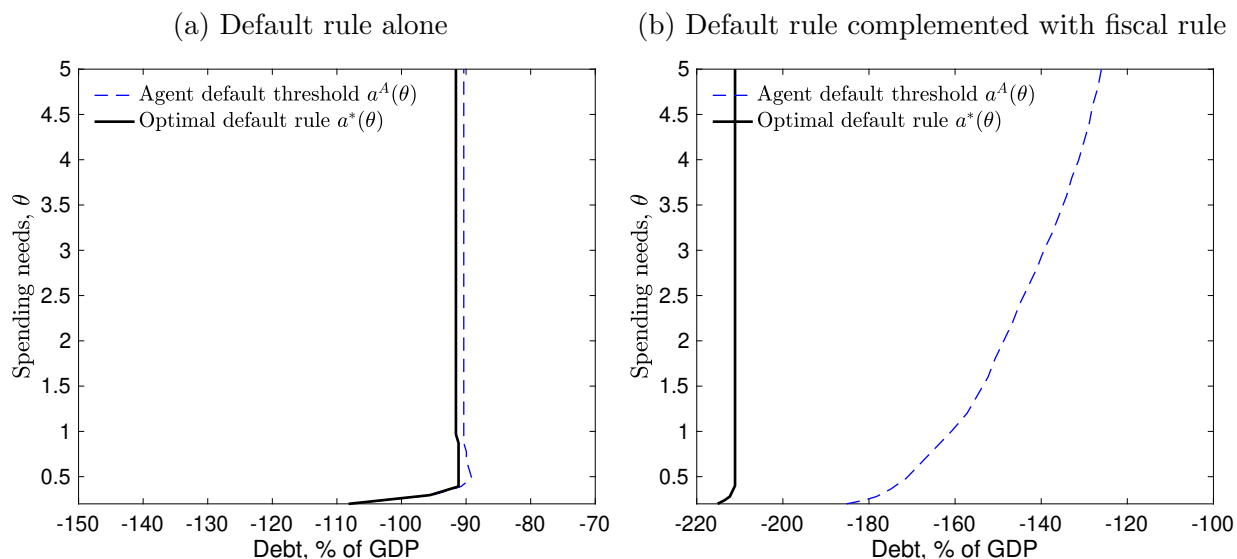
¹⁴The quantitative magnitude of the default risk premium is much larger than the empirical levels. The extension incorporating partial debt forgiveness in Section 4.1 would be able to match better the empirical magnitude.

does not impose any fiscal rule. The black solid line depicts this exercise in Panel (a) of Figure 5. The optimal default rule in this case has the pattern described in Proposition 1. It forbids default for debt levels below 92.5% of GDP and then coincides with the desired default thresholds of the governments. This small deviation from the governments' desired default thresholds is partially due to the interest-rate effect. By banning default, the planner makes sure that the nondefaulting types bear a lower burden by the debt services. The extent of intervention is so mild that one may even think that is not worth bothering. However, the situation is substantially different when the *default rule is complemented with the fiscal rule*.

In the second exercise, we compute the optimal default rule when the planner **also** imposes a fiscal rule, which is depicted in Panel (b) of Figure 5. To understand this figure, it is important to bear in mind that the fiscal rule drastically reduces the spending capacity of each government. Thus, the immediate effect of the fiscal rule is to build debt capacity. The financial markets are more confident that the country would repay its debt and thus willing to lend a larger amount. In the absence of a default rule, the governments would follow a default strategy depicted by the dashed blue line in Panel b) of Figure 5. This by itself expands the risk-free area from 0 to 90% of GDP when the fiscal rule is absent to 0-125% of GDP when the fiscal rule is imposed. In addition, it also increases the debt capacity from 115% of GDP to more than 180%. However, that is not the end of the optimal intervention. Given the fiscal rule, the planner also set different default rules, which is depicted in the black solid line. Now the extent of intervention is sizeable. The rule-writer forbids default of any debt level below 215% of GDP, while the area with discretionary default is drastically reduced. Note that the government would have a strong incentive to default for any debt level above 130%, but the default rule **optimally forbids it**.

Although the spirit of our calibration is not to accurately reflect any country in particular, the pattern that we called Greece-like generates some interesting analogies to the observations after the 2011 European debt crisis. In the intervention after the Greek default, the European troika imposed fiscal rules that were considered draconian from many viewpoints and validated a haircut that left Greece with a debt-to-GDP ratio barely short of 200%, which is considered unsustainable. In the light of our model, these highly criticized decisions are perfectly consistent with the optimal rules imposed by an unbiased planner given the country's political environment.

Figure 5: Optimal rules: Greece-like regime



6 Conclusions

Sovereign debt accumulation has long been a subject of controversial debate. Although the possibility of borrowing is accepted as an important tool to efficiently smooth adverse shocks, government debt is also widely regarded as exploited by self-interested governments for their own benefits. Hence, fiscal rules appear as fundamental components of every healthy democracy. This concern triggers hefty debates, especially when default is a possibility, reflecting the potential unsustainability of honoring past obligations, making the debate unavoidable. When the financial markets are incomplete, the possibility of defaulting on past obligations does not necessarily reflect an inefficiency; it could also be a welfare-improving tool. Thus, to analyze the need and optimality of fiscal rules, one must incorporate these three key elements: the need to smooth spending, political bias, and risk of default. In this paper, we have approached the problem based on this premise.

We have extended previous insights showing that limits on deficits or debt growth, together with discretion to respond to spending needs, are a necessary component of sound fiscal constraints in environments with risk of default. In addition, we have shown that an analogous principle applies to the default decision. Default decisions must also be regulated. Even though default rules are unusual, it does not mean that they are unnecessary. We have shown that sometimes, when debt is low, governments default too much too early, which calls for the need of banning default for low levels of debt. We also show that, especially when debt is large, governments default too little too late, so forcing default by imposing a “hard” debt limit would be optimal. Defaulting can be optimal, and governments concerned

only about the cost borne by their own administration inefficiently avoid it. Similarly, for intermediate levels of debt, defaulting is an optimal tool that can be welfare-improving. Regulating this decision whenever debt is neither too low nor too high is too costly, so governments should default at their discretion, using the information available to them.

We see this paper as a first step toward developing a theory that can help provide precise quantitative prescriptions for real-life case studies. At this stage, we have compromised on omitting important features of reality to derive precise theoretical prescriptions. Nevertheless, there are many dimensions in which this theory could be enriched. Among many other, the debt maturity structure and the endogeneity of political turnover appear as key elements that must be studied in follow-ups to this paper.

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A Proofs

A.1 Sequential Representation

We formulate the problem in sequential representation. When not in default, the value functions $w^n(\theta, b)$ and $v^n(\theta, b)$ satisfy

$$\begin{aligned} w^n(\theta, b) &= \int_0^T e^{-\rho t} \theta u(g_t) dt + e^{-\rho T} \beta \mathbb{E}[v(\theta', b_T)] \\ v^n(\theta, b) &= \int_0^T e^{-\rho t} \theta u(g_t) dt + e^{-\rho T} \mathbb{E}[v(\theta', b_T)], \end{aligned}$$

where $\{g_t\}_{t \geq 0}$ denotes the path of spending plan for the agent at state (θ, b) at time $t = 0$ and T denotes the time at which political turnover occurs for the first time.

Adjusting for the Poisson rate of the turnover shock, the value functions transform into

$$w^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} (\theta u(g_t) + \lambda \beta \mathbb{E}[v(\theta', b_t)]) dt \quad (33)$$

$$v^n(\theta, b) = \int_0^\infty e^{-(\rho+\lambda)t} (\theta u(g_t) + \lambda \mathbb{E}[v(\theta', b_t)]) dt. \quad (34)$$

Lemma 8. *Under the incentive-compatible spending plan $g(\theta, b)$,*

- i) The marginal values of asset satisfy $w_b^n(\theta, b) < v_b^n(\theta, b)$.*
- ii) The cross-partial of the agent's value function is bounded by:*

$$0 < w_{\theta b}^n(\theta, b) \leq \frac{1}{\theta} w_b^n(\theta, b), \forall \theta, b. \quad (35)$$

When $\beta > 0$, the inequality is strict: $w_{\theta b}^n(\theta, b) < \frac{1}{\theta} w_b^n(\theta, b)$.

Proof. To show item i), we differentiate equations (33) and (34) with respect to b :

$$\begin{aligned} w_b^n(\theta, b) &= \int_0^\infty e^{-(\rho+\lambda)t} \left(\theta u'(g_t) \frac{\partial g_t}{\partial b} + \lambda \beta \mathbb{E} \left[v_b(\theta', b_t) \frac{\partial b_t}{\partial b} \right] \right) dt \\ v_b^n(\theta, b) &= \int_0^\infty e^{-(\rho+\lambda)t} \left(\theta u'(g_t) \frac{\partial g_t}{\partial b} + \lambda \mathbb{E} \left[v_b(\theta', b_t) \frac{\partial b_t}{\partial b} \right] \right) dt. \end{aligned}$$

When the agent is endowed with more initial asset, the spending plan under a lower level of asset is feasible. However, we can improve the outcome by increasing spending at every point of time. Thus, it must be the case that $\frac{\partial g_t}{\partial b} > 0$. To afford higher future spending, it also must be that $\frac{\partial b_t}{\partial b} > 0$. Since $v_b(\theta, b) > 0$ and $\frac{\partial b_t}{\partial b} > 0$, it must be that $\mathbb{E} \left[v_b(\theta', b_t) \frac{\partial b_t}{\partial b} \right] > 0$. Thus we obtain that $w_b^n(\theta, b) < v_b^n(\theta, b)$.

For item ii), the envelope condition for equation (33) implies that

$$w_{\theta}^n(\theta, b) = \int_0^{\infty} e^{-(\rho+\lambda)t} u(g_t) dt.$$

Note that the condition above holds under the incentive-compatible spending. It also holds under the spending in the Markov equilibrium. Further differentiating with respect to b :

$$w_{\theta b}^n(\theta, b) = \int_0^{\infty} e^{-(\rho+\lambda)t} u'(g_t) \frac{\partial g_t}{\partial b} dt.$$

Since $u'(g_t) > 0$ and $\frac{\partial g_t}{\partial b} > 0$, we obtain that $w_{\theta b}^n(\theta, b) > 0$. Given that $\mathbb{E}[v_b(\theta', b_t) \frac{\partial b_t}{\partial b}] > 0$, we have $w_{\theta b}^n(\theta, b) \leq \frac{1}{\theta} w_b^n(\theta, b)$. It only holds with equality when there is extreme present bias, i.e., $\beta = 0$. \square

Lemma 8 holds under the incentive-compatible spending plan $g(\theta, b)$. It also holds under the spending in the Markov equilibrium.

A.2 Proof of Lemma 1

Differentiating the first-order condition (8) with respect to b :

$$\theta u''(g(\theta, b)) g_b(\theta, b) = w_{bb}(\theta, b). \quad (36)$$

Differentiating the HJB equations (11) for $w^n(\theta, b)$ with respect to b :

$$\begin{aligned} \left(\rho + \lambda - \frac{\partial(r(b)b)}{\partial b} \right) w_b^n(\theta, b) &= (\theta u'(g(\theta, b)) - w_b^n(\theta, b)) g_b(\theta, b) + \dot{b}(\theta, b) w_{bb}^n(\theta, b) \\ &+ \lambda \beta \mathbb{E}[v_b(\theta', b)]. \end{aligned} \quad (37)$$

Using the first-order condition (8) and equation (36) in the envelope condition (37) and incorporating that $\dot{b}(\theta, b) g_b(\theta, b) = \dot{g}(\theta, b)$, we obtain:

$$\left(\rho + \lambda - \frac{\partial(r(b)b)}{\partial b} \right) \theta u''(g(\theta, b)) = \dot{g}(\theta, b) \theta u''(g(\theta, b)) + \lambda \beta \mathbb{E}[v_b(\theta', b)].$$

Reorganizing the last equation to obtain:

$$\frac{\dot{g}(\theta, b)}{g(\theta, b)} = \left(\rho + \lambda - \frac{\partial(r(b)b)}{\partial b} \right) \frac{u'(g(\theta, b))}{g(\theta, b) u''(g(\theta, b))} - \lambda \beta \frac{\mathbb{E}[v_b(\theta', b)]}{\theta g(\theta, b) u''(g(\theta, b))}.$$

Consider a CRRA utility with risk aversion parameter denoted by γ . We have $\gamma = -\frac{gu''(g)}{u'(g)}$. Further, $\theta g(\theta, b) u''(g(\theta, b)) = -\gamma \theta u'(g(\theta, b)) = -\gamma w_b^n(\theta, b)$. We obtain equation (10). Since the cross-partial $w_{\theta b}^n(\theta, b) > 0$, $w_b^n(\theta, b)$ is strictly increasing in θ . Therefore the growth rate of spending $\frac{\dot{g}(\theta, b)}{g(\theta, b)}$ is decreasing in θ .

The effect due to the interest rate

$$\frac{\partial(r(b)b)}{\partial b} = r'(b)b + r(b) \geq r_f.$$

The larger β is, the larger the precautionary savings. When there is extreme present bias, i.e., $\beta = 0$, the growth rate of spending does not depend on type. As we show later that there is endogenous borrowing constraint, $\frac{\partial(r(b)b)}{\partial b} = r_f$, for $b > b^A$. All types would spend the same, and the path of spending grows at rate $\frac{1}{\gamma}(r_f - \rho - \lambda) < 0$.

A.3 Proof of Lemma 2

To understand how the agent's default threshold changes with type, we differentiate the indifference condition (9) with respect to θ and obtain:

$$\frac{\partial b^A(\theta)}{\partial \theta} = \frac{w_\theta^d(\theta) - w_\theta^n(\theta, b^A(\theta))}{w_b^n(\theta, b^A(\theta))}. \quad (38)$$

Differentiating the HJB equations (11) and (12) for $w^n(\theta, b)$ and $w^d(\theta)$ with respect to θ :

$$\begin{aligned} (\rho + \lambda) w_\theta^n(\theta, b) &= u(g(\theta, b)) + (\theta u'(g(\theta, b)) - w_b(\theta, b)) g_\theta(\theta, b) + \dot{b}(\theta, b) w_{b\theta}^n(\theta, b) \\ (\rho + \lambda) w_\theta^d(\theta) &= u(\kappa\tau) + \phi(w_\theta^n(\theta, 0) - w_\theta^d(\theta)). \end{aligned}$$

In the Markov equilibrium, the first-order condition (8) for spending holds. In the incentive-compatible allocation, the IC constraint (43) holds. In either case, the first of the two equations above reduces to

$$(\rho + \lambda) w_\theta^n(\theta, b) = u(g(\theta, b)) + \dot{b}(\theta, b) w_{b\theta}^n(\theta, b).$$

Substituting the expressions above in equation (38):

$$\frac{\partial b^A(\theta)}{\partial \theta} = \frac{1}{\rho + \lambda} \frac{\phi(w_\theta^n(\theta, 0) - w_\theta^d(\theta)) - \left(u(g(\theta, b^A(\theta))) - u(\kappa\tau) + \dot{b}(\theta, b^A(\theta)) w_{b\theta}^n(\theta, b^A(\theta)) \right)}{w_b^n(\theta, b^A(\theta))}.$$

At the discretionary default threshold, the indifference condition (9) implies that

$$\begin{aligned} & \theta (u (g (\theta, b^A (\theta))) - u (\kappa\tau)) + \dot{b} (\theta, b^A (\theta)) w_b^n (\theta, b^A (\theta)) \\ & = -\lambda\beta (\mathbb{E} [v (\theta', b^A (\theta))] - \mathbb{E} [v^d (\theta')]) + \phi (w^n (\theta, 0) - w^d (\theta)). \end{aligned} \quad (39)$$

Consider when the exclusion from the financial market is permanent, i.e., $\phi = 0$. The difference in the expected continuation values:

$$\mathbb{E} [v (\theta, b)] - \mathbb{E} [v^d (\theta)] = \mathbb{E} [(v^n (\theta, b) - v^d (\theta)) (1 - \delta (\theta, b))] \geq 0.$$

This is because, at the upper bound of debt, all types default: $\delta (\theta', \underline{b}^A) = 1, \forall \theta$. Therefore, $\mathbb{E} [v (\theta, \underline{b}^A)] - \mathbb{E} [v^d (\theta)] = 0$. Since $\mathbb{E} [v (\theta, b)]$ is strictly increasing in b , we obtain that $\mathbb{E} [v (\theta, b)] - \mathbb{E} [v^d (\theta)] > 0, \forall b > \underline{b}^A$.

According to Lemma 8, we have $w_{b\theta}^n (\theta, b) \leq \frac{1}{\theta} w_b^n (\theta, b)$. In addition, at the default threshold, $\dot{b} (\theta, b^A (\theta)) \geq 0$. Therefore,

$$\begin{aligned} & u (g (\theta, b^A (\theta))) - u (\kappa\tau) + \dot{b} (\theta, b^A (\theta)) w_{b\theta}^n (\theta, b^A (\theta)) \\ & \leq u (g (\theta, b^A (\theta))) - u (\kappa\tau) + \dot{b} (\theta, b^A (\theta)) \frac{1}{\theta} w_b^n (\theta, b^A (\theta)) \leq \phi \frac{1}{\theta} (w^n (\theta, 0) - w^d (\theta)). \end{aligned}$$

Therefore, the default threshold is increasing in type: $\frac{\partial b^A (\theta)}{\partial \theta} \geq 0$.

When there exists savers, the lowest type $\underline{\theta}$ must be a saving type. Given that the default threshold is increasing in type, we have that the debt lower bound $\underline{b}^A = b^A (\underline{\theta})$. Then $w_{b\theta}^n (\underline{\theta}, b) < \frac{1}{\underline{\theta}} w_b^n (\underline{\theta}, b)$ and $\dot{b} (\underline{\theta}, \underline{b}^A) > 0$. For the lowest type, the first inequality in the equation above is strict. For all other types, the second inequality is strict. Therefore the default threshold is strictly increasing everywhere: $\frac{\partial b^A (\theta)}{\partial \theta} > 0$, for all θ .

A.4 Proof of Lemma 3

Consider an economy with an endogenous borrowing limit, $b^A (\theta) = b^A$. The highest type $\bar{\theta}$ is always a borrowing type at any level of debt, $\dot{b} (\bar{\theta}, b) \leq 0$. Therefore, at the borrowing limit, the highest type has zero net saving, $\dot{b} (\bar{\theta}, b^A) = 0$. According to equation (39), the borrowing limit satisfies

$$\bar{\theta} (u (r_f b^A + \tau) - u (\kappa\tau)) = \phi (w^n (\bar{\theta}, 0) - w^d (\bar{\theta})),$$

which implies that

$$b^A = \frac{1}{r_f} \left(u^{-1} \left(u(\kappa\tau) + \phi \frac{w^n(\bar{\theta}, 0) - w^d(\bar{\theta})}{\bar{\theta}} \right) - \tau \right). \quad (40)$$

The equilibrium interest rate is discontinuous at the borrowing limit,

$$r(b) = \begin{cases} r_f, & \text{if } b \geq b^A \\ \infty, & \text{if } b < b^A. \end{cases}$$

Now suppose there is a type that would like to default at a slightly higher debt level $b < b^A$. At that debt level, the market would charge an interest rate $r(b) = r_f + \lambda$. If it leads to an interest payment above the tax revenue, even if the government incurs zero spending, it would not be able to cover the interest payment:

$$-(r_f + \lambda)b > -(r_f + \lambda)b^A \geq \tau.$$

Thus it would not be possible for any type to borrow more. When the tax revenue is just able to cover the interest payment, we obtain an expression for $\bar{\lambda}$:

$$(r_f + \bar{\lambda}) \frac{1}{r_f} \left(u^{-1} \left(u(\kappa\tau) + \phi \frac{w^n(\bar{\theta}, 0) - w^d(\bar{\theta})}{\bar{\theta}} \right) - \tau \right) + \tau = 0.$$

When the financial exclusion is permanent, i.e., $\phi = 0$, the debt capacity is $b^A = \frac{1}{r_f}(\kappa - 1)\tau$. The threshold turnover rate has a simple analytical expression:

$$\lambda \geq \bar{\lambda} = \frac{r_f \kappa}{1 - \kappa}.$$

A.5 Proof of Proposition 1

Given a mechanism \mathcal{M} , the agent chooses the report x to maximize its value:

$$\max_x w(x, \theta, b; \mathcal{M}), \forall \theta, b.$$

Default rule. We start by examining the default rule $\delta(\theta, b)$. To do so, we fix a candidate spending rule $g(\theta, b)$.

First, we show that, at debt levels where some types default and others do not, i.e., $b \in (\underline{b}, \bar{b})$, the only possible default rule that can induce truth-telling is one that allows for

discretionary default, $\delta(\theta, b) = \delta^A(\theta, b)$. For a given level of debt b , we partition the set of reports into two subsets: the nondefault set and the default set,

$$\Theta^n(b) = \{\theta \in \Theta : \delta(\theta, b) = 0\} \quad \text{and} \quad \Theta^d(b) = \{\theta \in \Theta : \delta(\theta, b) = 1\}.$$

If the principal wishes to implement partial default, $\mathbb{E}[\delta(\theta, b)] \in (0, 1)$, the sets $\Theta^n(b)$ and $\Theta^d(b)$ are both nonempty. The agent will report in the nondefault set if and only if

$$\max_{x \in \Theta^n(b)} w^n(x, \theta, b; \mathcal{M}) > \max_{x \in \Theta^d(b)} w^d(x, \theta; \mathcal{M}).$$

The binary nature of the default choice makes it impossible for the principal to alter the agent's default behavior. The types that can obtain a higher value defaulting would pretend to be in the default set Θ^d . Likewise, the types that can obtain a higher value not defaulting would pretend to be the type that maximizes its value in the nondefault set Θ^n .

Given we consider truth-telling mechanisms, if the agent is told not to default according to its report, its type must be in the nondefault set, $x = \theta \in \Theta^n(b)$. Imposing truth-telling, the IC constraint can be written as $\delta(\theta, b) = 0$ if and only if $w^n(\theta, b) \geq w^d(\theta)$. And it is only relevant in the partial default area.

Next, we show that, under the optimal rule, the default decision $\delta(\theta, b)$ must be decreasing in b . We set up the Lagrangian for the problem in (13). Let $\psi(\theta, b)$ be the Lagrange multiplier for the truth-telling constraint for type θ at debt b .

$$\mathcal{L} = \int_{\underline{\theta}}^{\bar{\theta}} (v^n(\theta, b)(1 - \delta(\theta, b)) + v^d(\theta)\delta(\theta, b) + \psi(\theta, b)(w^n(\theta, b) - w^d(\theta))) dF(\theta).$$

If type θ does not default at debt b , in comparison to default, the Lagrangian changes by

$$\left[v^n(\theta, b) - v^d(\theta) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta', b)}{\partial r(b)} (1 - \delta(\theta', b)) dF(\theta') \right] f(\theta) + \psi(\theta, b)(w^n(\theta, b) - w^d(\theta)). \quad (41)$$

Since the IC constraint is only relevant in the partial default area, $\psi(\theta, b) = 0$ in the zero default area and the full default area. In the expression in (41), the third term $\lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta', b)}{\partial r(b)} (1 - \delta(\theta', b)) dF(\theta')$ captures the interest-rate effect of default decision of θ on other types, which is why it is multiplied by the density $f(\theta)$. However, the value of this term is unaffected by the default choice for this specific type. Fixing the default choices for all other types, $\delta(\theta', b)$, $\forall \theta' \neq \theta \in \Theta$, $\forall b$, the default choice for type θ does not affect the interest rates $r(b)$. Therefore, the expression in (41) is strictly increasing in b . Thus we conclude that the optimal default decision $\delta(\theta, b)$ is decreasing in b . There exists a unique

threshold $b^*(\theta)$ such that

$$\delta(\theta, b) = \begin{cases} 1, & \text{if } b < b^*(\theta) \\ 0, & \text{if } b \geq b^*(\theta). \end{cases}$$

This in turn implies that the default probability $\mathbb{E}[\delta(\theta, b)]$ is decreasing in the amount of assets b , so is the interest rate $r(b)$.

Further, at the natural limit $-\frac{\tau}{r_f}$, the principal would always want to default. When the debt position is zero, the principal would never want to default. There exists an upper bound of debt $\underline{b} > \frac{\tau}{r_f}$ such that beyond this debt level there is full default, $\delta(\theta, b) = 1, \forall \theta$, for all $b \leq \underline{b}$. There also exists a lower bound of debt $\bar{b} \leq 0$ such that within this debt level there is zero default, $\delta(\theta, b) = 0, \forall \theta$, for all $b \geq \bar{b}$. The following relations hold: $-\frac{\tau}{r_f} < \underline{b} \leq \bar{b} \leq 0$. In the intermediate debt level, $\forall b \in (\underline{b}, \bar{b})$, the principal allows for discretionary default: $\delta(\theta, b) = \delta^A(\theta, b), \forall \theta$.

Spending rule. In the nondefault area, the optimality condition with respect to the report:

$$w_x^n(x, \theta, b; \mathcal{M}) = 0, \forall b \geq b^*(\theta), \forall \theta.$$

Differentiating the optimal condition with respect to b , we obtain that $w_{bx}^n(x, \theta, b) = 0$. The condition above holds with strict equality. Otherwise if $\delta(x, b) = 1$,

$$w_x^d(x, \theta; \mathcal{M}) = \frac{\phi}{\rho + \phi + \lambda} w_x^n(x, \theta, 0; \mathcal{M}) = 0.$$

When the debt position is zero, the agent will never default. If the agent reports truthfully when its debt position is zero, the agent will report truthfully when in default.

Without loss of generality, we restrict to spending $g(\theta, b)$ that is continuously differentiable almost everywhere. Differentiating the HJB equation (11) with respect to x : for $b \geq b^*(x)$,

$$(\rho + \lambda) w_x^n(x, \theta, b) = (\theta u'(g(x, b)) - w_b^n(x, \theta, b)) g_x(x, b) + \dot{b}(x, b) w_{bx}^n(x, \theta, b). \quad (42)$$

There is no direct effect of $b^*(x)$ and any indirect effect is taken care of by $w_{bx}^n(x, \theta, b)$, which we have shown is zero.

$$(\rho + \lambda) w_x^n(x, \theta, b) = (\theta u'(g(x, b)) - w_b^n(x, \theta, b)) g_x(x, b) = 0.$$

Imposing truth-telling, $x = \theta$, the IC constraint for spending in equation (42) becomes

$$(\theta u'(g(\theta, b)) - w_b^n(\theta, b)) g_\theta(\theta, b) = 0. \quad (43)$$

We can see from equation (43) that there is limited scope for intervention: the first component is the same as without intervention, and the second term implies that intervention can only constrain changes in spending. Thus, same as in the economy without default, the optimal intervention either gives flexibility, allowing agents to spend at their discretion according to their first-order condition (8) or setting a rule where $g_\theta(\theta, b) = 0$ holds.

Before proceeding further, we show that the optimal spending is monotone.

Lemma 9. *The optimal spending rule $g(\theta, b)$ is monotonically increasing in θ :*

$$g_\theta(\theta, b) \geq 0.$$

Proof. We only need to show that, in the discretionary spending area $g_\theta(\theta, b) \geq 0$, since in the area with rules $g_\theta(\theta, b) = 0$. In the discretionary spending area, the first-order condition (8) for spending holds. Differentiating it with respect θ :

$$u'(g(\theta, b)) + \theta u''(g(\theta, b)) g_\theta(\theta, b) = w_{\theta b}^n(\theta, b). \quad (44)$$

Combining the expressions in equations (8) and (44), we obtain that

$$\theta u''(g(\theta, b)) g_\theta(\theta, b) = w_{\theta b}^n(\theta, b) - \frac{1}{\theta} w_b^n(\theta, b). \quad (45)$$

According to Lemma 8, the right-hand side of equation (45) is negative. Further, the utility function is strictly concave $u''(\cdot) < 0$. Thus it must be that spending is increasing in type $g_\theta(\theta, b) \geq 0$. \square

Given the monotonicity result for spending in Lemma 9, it implies that incentive-compatible allocations features a spending threshold, denote by $\theta^s(b)$. All types with $\theta \leq \theta^s(b)$ have flexibility and all types $\theta > \theta^s(b)$ are bunched and spend the same as type $\theta^s(b)$.

A.6 Proof of Lemma 4

Lemma 10. *For any threshold $\theta^s(b)$, in the area with rules, the marginal values of asset satisfy*

$$w_b^n(\theta, b) = \beta v_b^n\left(\frac{\theta}{\beta}, b\right), \quad \forall \theta \geq \theta^s(b), \forall b. \quad (46)$$

The value functions are affine in θ :

$$\begin{aligned} v^n(\theta, b) &= v_1(b) + \theta v_2(b) \\ w^n(\theta, b) &= \beta v_1(b) + \theta v_2(b). \end{aligned}$$

Proof. Differentiating the HJB equation (4) for $v^n(\theta, b)$ with respect to b :

$$\begin{aligned} \left(\rho + \lambda - \frac{\partial(r(b)b)}{\partial b} \right) v_b^n(\theta, b) &= (\theta u'(g(\theta, b)) - v_b^n(\theta, b)) g_b(\theta, b) + \dot{b}(\theta, b) v_{bb}^n(\theta, b) \\ &+ \lambda \mathbb{E}[v_b(\theta', b)]. \end{aligned} \quad (47)$$

Consider a spending threshold θ^s . For a type $\theta > \theta^s$, its spending is bunched to $g(\theta^s, b)$ and its debt position evolves according to $\dot{b}(\theta^s, b)$. The same applies to type $\frac{\theta}{\beta}$. Evaluating equation (47) at $v_b^n\left(\frac{\theta}{\beta}, b\right)$ and multiplying both sides by β , comparing with equation (37), one can see that (46) holds.¹⁵

We guess and verify that the value functions are affine in θ . Substituting the guesses in the HJB equations and (11) and (4), we obtain the following ordinary-differential equations:

$$\begin{aligned} (\rho + \lambda) v_1(b) &= \dot{b}(\theta^s, b) v_1'(b) + \lambda \mathbb{E}[v(\theta', b)] \\ (\rho + \lambda) v_2(b) &= u(g(\theta^s, b)) v_1'(b) + \dot{b}(\theta^s, b) v_2'(b). \end{aligned}$$

□

Consider an economy in which the default punishment is extreme $\kappa = 0$. The agents would never want to default, and thus they can borrow up to the natural debt limit $\frac{\tau}{r_f}$. In problem (13), we only need to design the spending rule $g(\theta, b)$, or equivalently the spending threshold $\theta^s(b)$. Formally, the planner chooses θ^s to maximize

$$\mathbb{E}[v^n(\theta, b)] = \int_{\underline{\theta}}^{\theta^s} v^n(\theta, b) dF(\theta) + \int_{\theta^s}^{\bar{\theta}} v^n(\theta, b) dF(\theta).$$

subject to equation (15).

The optimality condition with respect to θ^s :

$$\frac{\partial \mathbb{E}[v^n(\theta, b)]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\bar{\theta}} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0. \quad (48)$$

¹⁵For $\theta > \beta \bar{\theta}$, the corresponding value of θ/β is outside the domain Θ . In this case, we can extend the v_b^n to be also defined in the range $(\bar{\theta}, \bar{\theta}/\beta]$, where the density is zero.

Since the optimality condition (48) holds for all b , the cross-partial $\frac{\partial \mathbb{E}[v_b^n(\theta, b)]}{\partial \theta^s} = 0$.

We first examine types $\theta \leq \theta^s$. According to the HJB equations, we have

$$\begin{aligned} (\rho + \lambda) \frac{\partial w^n(\theta, b)}{\partial \theta^s} &= \dot{b}(\theta, b) \frac{\partial w_b^n(\theta, b)}{\partial \theta^s} + \lambda \beta \frac{\partial \mathbb{E}[v_b(\theta, b)]}{\partial \theta^s} \\ (\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} &= (\theta u'(g(\theta, b)) - v_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial \theta^s} + \dot{b}(\theta, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} + \lambda \frac{\partial \mathbb{E}[v_b^n(\theta, b)]}{\partial \theta^s}. \end{aligned}$$

We guess and verify that $\frac{\partial w^n(\theta, b)}{\partial \theta^s} = \frac{\partial w_b^n(\theta, b)}{\partial \theta^s} = 0$, which implies that $\frac{\partial g(\theta, b)}{\partial \theta^s} = 0$. We also guess and verify that $\frac{\partial v^n(\theta, b)}{\partial \theta^s} = \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} = 0$. Therefore the optimality condition (48) simplifies to

$$\int_{\theta^s}^{\bar{\theta}} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0,$$

Now we examine types $\theta > \theta^s$, we have:

$$(\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} = (\theta u'(g(\theta^s, b)) - v_b^n(\theta, b)) g_\theta(\theta^s, b) + \dot{b}(\theta^s, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s}.$$

The optimality condition turns into

$$g_\theta(\theta^s, b) \int_{\theta^s}^{\bar{\theta}} (\theta u'(g(\theta^s, b)) - v_b^n(\theta, b)) dF(\theta) + \dot{b}(\theta^s, b) \int_{\theta^s}^{\bar{\theta}} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0.$$

Since $g_\theta(\theta^s, b) > 0$ and the cross-partial $\int_{\theta^s}^{\bar{\theta}} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) = 0$, we obtain:

$$\int_{\theta^s}^{\bar{\theta}} (\theta u'(g(\theta^s, b)) - v_b^n(\theta, b)) dF(\theta) = 0$$

Assessing the marginal utility for spending at the threshold according to the first-order condition (8) and using the affine result in Lemma 10, we have $\theta u'(g(\theta^s, b)) = \frac{\theta}{\theta^s} w_b^n(\theta^s, b) = \frac{\theta}{\theta^s} \beta v_1'(b) + \theta v_2'(b)$ and $v_b^n(\theta, b) = v_1'(b) + \theta v_2'(b)$. Replacing in the last equation, we have

$$\int_{\theta^s}^{\bar{\theta}} (\beta \theta - \theta^s) dF(\theta) = 0,$$

which can be rewritten as equation (17). It shows that the spending threshold is independent of b . Assumption 1 ensures that $\beta \mathbb{E}[\theta | \theta > \theta^s] - \theta^s$ is strictly decreasing in θ^s . Given that $\beta \bar{\theta} - \bar{\theta} \leq 0$, if $\beta > \underline{\theta}$, equation (17) has a unique root.

A.7 Proof of Proposition 2

At any debt level $b < \underline{b}$, all types default, i.e., $\delta(\theta, b) = 1$. There is no spending rule to be made since all types are in autarky. At any debt level $b > \bar{b}$, no type defaults, i.e., $\delta(\theta, b) = 0$. The interest rate $r(b) = r_f$, and equation (18) holds trivially. Now we consider a debt level $b \in [\underline{b}, \bar{b}]$.

When $\phi = 0$, Lemma 2 implies a monotone increasing default threshold in type, $b^A(\theta)$. Therefore there exists a default threshold type θ^d . Given the default rule, it is without loss of generality to consider the bunching threshold in the nondefault area, $\theta^s \leq \theta^d$. The default value is unaffected by the spending threshold: $\frac{\partial v^d(\theta)}{\partial \theta^s} = 0$.

The maximization objective in (13) can be rewritten as

$$\mathbb{E}[v(\theta, b)] = \int_{\underline{\theta}}^{\theta^s} v^n(\theta, b) dF(\theta) + \int_{\theta^s}^{\theta^d} v^n(\theta, b) dF(\theta) + \int_{\theta^d}^{\bar{\theta}} v^d(\theta) dF(\theta).$$

The optimality condition with respect to θ^s is

$$\frac{\partial \mathbb{E}[v(\theta, b)]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\theta^d} \frac{\partial v^n(\theta, b)}{\partial \theta^s} dF(\theta) + (v^n(\theta^d, b) - v^d(\theta^d)) f(\theta^d) \frac{\partial \theta^d}{\partial \theta^s} = 0.$$

Since the condition above holds for all b , the cross partial is also zero:

$$\frac{\partial \mathbb{E}[v_b(\theta, b)]}{\partial \theta^s} = \int_{\underline{\theta}}^{\theta^s} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) + \int_{\theta^s}^{\theta^d} \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} dF(\theta) + v_b^n(\theta^d, b) f(\theta^d) \frac{\partial \theta^d}{\partial \theta^s} = 0.$$

For types below the spending threshold $\theta < \theta^s$, we have

$$(\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} = (\theta u'(g(\theta, b)) - v_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial \theta^s} + \dot{b}(\theta, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} + \frac{\partial r(b)}{\partial \theta^s} b v_b(\theta, b).$$

For types above the spending threshold $\theta \geq \theta^s$, we have

$$(\rho + \lambda) \frac{\partial v^n(\theta, b)}{\partial \theta^s} = (\theta u'(g(\theta^s, b)) - v_b^n(\theta, b)) \frac{\partial g(\theta^s, b)}{\partial \theta^s} + \dot{b}(\theta^s, b) \frac{\partial v_b^n(\theta, b)}{\partial \theta^s} + \frac{\partial r(b)}{\partial \theta^s} b v_b(\theta, b).$$

At the default threshold, given that it must be that $\dot{b}(\theta^s, b) = 0$, we have

$$(\rho + \lambda) (v^n(\theta^d, b) - v^d(\theta^d)) = \theta^d (u(g(\theta^s, b)) - u(\kappa\tau)) + \lambda (\mathbb{E}[v(\theta', b)] - \mathbb{E}[v(\theta')]).$$

Finally, the interest rate adjusts to the spending threshold according to

$$\frac{\partial r(b)}{\partial \theta^s} = -\lambda f(\theta^d) \frac{\partial \theta^d}{\partial \theta^s}.$$

Substituting the equations above into the optimality condition:

$$\begin{aligned} \int_{\theta^s}^{\theta^d} (\theta u'(g(\theta^s, b)) - v_b^n(\theta, b)) dF(\theta) \frac{\partial g(\theta^s, b)}{\partial \theta^s} + \left\{ \int_{\underline{\theta}}^{\theta^s} (\theta u'(g(\theta, b)) - v_b^n(\theta, b)) \frac{\partial g(\theta, b)}{\partial r(b)} dF(\theta) \right. \\ \left. + b \mathbb{E}[v_b(\theta, b)] + \int_{\underline{\theta}}^{\theta^s} (g(\theta^s, b) - g(\theta, b)) \frac{\partial v_b^n(\theta, b)}{\partial r(b)} dF(\theta) \right. \\ \left. - \left[\frac{1}{\lambda} \theta^d (u(g(\theta^s, b)) - u(\kappa\tau)) + \mathbb{E}[v(\theta', b)] - \mathbb{E}[v(\theta')] \right] \right\} \frac{\partial r(b)}{\partial \theta^s} = 0. \end{aligned}$$

Using the derivations from the previous proof in Section A.6, we obtain equation (18), where

$$\chi(b) = \frac{\theta^s (F(\theta^d) - F(\theta^s))}{v_1'(b) \frac{\partial g(\theta^s, b)}{\partial \theta^s}}.$$

A.8 Proof of Lemma 5

We build on the arguments in the proof for Proposition 1. When there is no information friction, the IC constraint for default becomes irrelevant everywhere. The expression in (41) immediately implies equation (19).

the lower and upper bounds \underline{b}^P and \bar{b}^P . When $\phi = 0$, default decision $\delta(\theta, b)$ is monotonously increasing in θ . If the lowest type should default,

$$v^n(\underline{\theta}, \underline{b}^P) = v^d(\underline{\theta}) \quad (49)$$

If the highest type should not default,

$$v^n(\bar{\theta}, \bar{b}^P) + \lambda \int_{\underline{\theta}}^{\bar{\theta}} \frac{\partial v^n(\theta, \bar{b}^P)}{\partial r(\bar{b}^P)} dF(\theta) = v^d(\bar{\theta}). \quad (50)$$

A.9 Proof of Proposition 3

For the planner, the borrowing limit it would like to implement is such that

$$\bar{\theta} (u(r_f \bar{b}^* + \tau) - u(\kappa\tau)) = \phi (v^n(\bar{\theta}, 0) - v^d(\bar{\theta})),$$

which implies that

$$b^* = \frac{1}{r_f} \left(u^{-1} \left(u(\kappa\tau) + \phi \frac{v^n(\bar{\theta}, 0) - v^d(\bar{\theta})}{\bar{\theta}} \right) - \tau \right). \quad (51)$$

Taking the difference between the HJB equations for the nondefault values and the default values and evaluating at $b = 0$, we obtain that

$$\begin{aligned} (\rho + \lambda + \phi) (w^n(\theta, 0) - w^d(\theta)) &= \theta (u(g(\theta, 0)) - u(\kappa\tau)) + \dot{b}(\theta, 0) w_b^n(\theta, 0) \\ &\quad + \lambda\beta (\mathbb{E}[v(\theta', 0)] - \mathbb{E}[v^d(\theta')]) \\ (\rho + \lambda + \phi) (v^n(\theta, 0) - v^d(\theta)) &= \theta (u(g(\theta, 0)) - u(\kappa\tau)) + \dot{b}(\theta, 0) v_b^n(\theta, 0) \\ &\quad + \lambda (\mathbb{E}[v(\theta', 0)] - \mathbb{E}[v^d(\theta')]). \end{aligned}$$

Define an auxiliary expression $\tilde{v}^n(\theta, 0) - \tilde{v}^d(\theta, 0)$ that satisfies below:

$$\begin{aligned} (\rho + \lambda + \phi) (\tilde{v}^n(\theta, 0) - \tilde{v}^d(\theta)) &= \theta (u(g(\theta, 0)) - u(\kappa\tau)) + \dot{b}(\theta, 0) \tilde{v}_b^n(\theta, 0) \\ &\quad + \lambda\beta (\mathbb{E}[v(\theta', 0)] - \mathbb{E}[v^d(\theta')]). \end{aligned}$$

It immediately follows that

$$v^n(\theta, 0) - v^d(\theta) \geq \tilde{v}^n(\theta, 0) - \tilde{v}^d(\theta) = w^n(\theta, 0) - w^d(\theta). \quad (52)$$

It is also obvious from the sequential representation why $v^n(\theta, 0) - v^d(\theta) \geq w^n(\theta, 0) - w^d(\theta)$: starting at zero debt position, given any sequence of spending, the streams of utility from spending before the taste shock arrives are identical, the only difference is that the government discounts the future utilities after the shock by a factor β . Hence $b^* \geq b^A$.

In the following cases, the default incentives between the agent and the principal are exactly aligned. When the financial exclusion is permanent, i.e., $\phi = 0$, the borrowing limits in equations (40) and (51) simplify to

$$b^* = b^A = \frac{1}{r_f} (\kappa - 1) \tau.$$

When there is zero cost of default, either because there is no revenue loss, i.e., $\kappa = 1$, or financial reaccess happens instantly, i.e., $\phi = \infty$, the borrowing limits in equations (40) and (51) becomes

$$b^* = b^A = 0.$$

However, in general, in the presence of present bias $\beta < 1$, there is a gap in the value

financial reaccess for the agent and the planner, $v^n(\theta, 0) - v^d(\theta) > w^n(\theta, 0) - w^d(\theta)$. If $\kappa < 1$ and $0 < \phi < \infty$, the planner would prefer to strictly default earlier,

$$b^* > b^A.$$

A.10 Proof of Proposition 4

The highest type $\bar{\theta}$ is always borrowing and accumulating debt. The question is whether the lowest type $\underline{\theta}$ is a saver or a borrower. Suppose that there is an endogenous borrowing limit. At the borrowing limit b^A , which is characterized in equation (40), all types would spend $r_f b^A + \tau$ and $\dot{b}(\theta, b^A) = 0$. The interest rate is right-differentiable: $\frac{\partial(r(b)b)}{\partial b} = -r_f$. The envelope condition (37) becomes

$$w_b^n(\theta, b^A) = \frac{\lambda\beta}{\rho + \lambda - r_f} \mathbb{E}[v_b(\theta', b^A)], \forall \theta.$$

If condition (21) is satisfied, spending amount $r_f b^A + \tau$ is not optimal for the lowest type $\underline{\theta}$ at the borrowing constraint. Instead, it has incentive to deviate and save at least a little bit.

A.11 Computation of Interest Rate

To compute the interest rate with partial default, we need the recovery value of the defaulted debt for each incumbent θ . However, conditional on repayment, the payment is independent of θ , the type only matters to determine when the incumbent would reaccess the financial markets, i.e., whether $\delta_d(\theta, b^h)$ is 0 or 1. Let $R^1(b^h)$ be the value of the debt that is currently in default, and there is government for which $\delta_d(\theta, b^h) = 1$. The value of this debt satisfies the recursion:

$$(r_f + \lambda)R^1(b^h) = 0 + \lambda\mathbb{E}[R(b^h)]$$

This happens because the flow payment is zero, and even if the government has the chance to return, it will not do it.

Let $R^0(b^h)$ be the value of the debt that is currently in default and there is government for which $\delta_d(\theta, b^h) = 0$. This government if it had the chance it would return to the financial markets. Thus, the value satisfies the recursion:

$$(r_f + \lambda + \phi)R^0(b^h) = 0 + \phi b^h + \lambda\mathbb{E}[R(b^h)],$$

where $R(b^h) = \delta_d(\theta, b^h)R^1(b^h) + (1 - \delta_d(\theta, b^h))R^0(b^h)$ is the ex ante expected value of the

debt, which satisfies:

$$\mathbb{E}[R(b^h)] = \mathbb{E}[\delta_d(\theta, b^h)]R^1(b^h) + (1 - \mathbb{E}[\delta_d(\theta, b^h)])R^0(b^h).$$

Combining all the equations, we obtain:

$$\mathbb{E}[R(b^h)] = \mathbb{E}[\delta_d(\theta, b^h)]\frac{\lambda\mathbb{E}[R(b^h)]}{r_f + \lambda} + (1 - \mathbb{E}[\delta_d(\theta, b^h)])\frac{(\phi b^h + \lambda\mathbb{E}[R(b^h)])}{r_f + \lambda + \phi}.$$

The solution is

$$\mathbb{E}[R(b^h)] = \frac{b^h(1 - \mathbb{E}[\delta_d(\theta, b^h)])\phi(\lambda + r_f)}{\lambda\phi(1 - \mathbb{E}[\delta_d(\theta, b^h)]) + r_f^2 + \lambda r_f + r_f\phi},$$

which confirms that the recovery value is linear in the amount defaulted. Then we have:

$$R^1(b^h) = \frac{b^h(1 - \mathbb{E}[\delta_d(\theta, b^h)])\lambda\phi}{\lambda\phi(1 - \mathbb{E}[\delta_d(\theta, b^h)]) + r_f(r_f + \lambda + \phi)}. \quad (53)$$

The last equation makes clear that the recovery value is linear in b^h . Thus, setting $\alpha = 1$, so that the first appearance of b^h in (53) is replaced by b , but keeping the dependency of δ_d on b^h we obtain (27), where the first line is R^1/b^h and the second R^0/b^h .

B Data

B.1 Data Sources

We obtain the data on debt from IMF's Global Debt Database. Government debt is measured using central government debt (percentage of GDP). The time series is available for Germany during year 1961-2018 and for Greece, Italy, and Argentina during year 1950-2018. There is an alternative measure using general government debt (percentage of GDP). The two measures have small discrepancies and generate remarkably similar debt growth profiles. We use the growth paths before 1993, the year the Maastricht Treaty came into force. Table 2 reports the average and standard deviation of debt growth, for the sub-periods pre-1993 and post-1993. Debt accumulation slowed down by about a half after Maastricht.

We obtain the data on government revenue and expenditures from the annual macro-economic database of the European Commission's Directorate General for Economic and Financial Affairs (AMECO). Government revenue is defined as total general government revenue (percentage of GDP). We use the available series for Germany during year 1991-2019 and for Greece and Italy during year 1995-2019. As shown in Table 2, the government sizes have remained stable.

Table 2: Data moments

	Germany	Greece	Italy	Argentina
Debt growth, % of GDP (pre-1993)				
Mean	0.5%	1.7%	1.7%	0.2%
Std dev	1.0%	3.4%	2.6%	15.4%
Debt growth, % of GDP (post-1993)				
Mean	0.6%	4.3%	1.2%	2.2%
Std dev	2.6%	9.6%	3.8%	23.6%
Government revenue, % of GDP				
Mean	44.9%	42.3%	45.5%	
Std dev	1.0%	4.3%	1.4%	
Government duration				
Mean (prime minister change)	7.53	1.29	1.61	
Mean (all changes)	1.81	0.93	0.92	

We obtain the Information on government duration from the Party Government Data Set. [Felli \(2020\)](#) uses data on average government duration to calibrate the political turnover rate λ . In [Table 2](#), we report two measures of government duration. One captures only prime minister change. Another captures all changes, including changes in prime minister, party ideology, party name, and prime minister ideology. From these statistics, we can see that Germany has much lower political turnover than Greece and Italy with all measurements.